ON THE FATIQUE OF LAMINATED COMPOSITES WITH ADHESIVE BONDS

IULIAN GIRIP , VALERICA MOSNEGUTU

Abstract. The fatigue of laminated composites with adhesive bonds is studied in this paper in relation with the fatigue process. Discrete and continuous damage approaches are presented together with a fatigue criterion based on the effective plastic work. The distribution of the inelastic dissipated energy shows that there is an important loss of inelastic energy in the bonding zone. That means that the bonding zone would be that fails.

Key words: adhesive bonding, fatique criteria.

1. INTRODUCTION

Adhesive bonding ties up two materials with the help of an adhesive. This combination offers important advantages over other blending methods such as welding and riveting [1–5]. Composite structures with adhesive joins are used in the aircraft, helicopter and satellite industry, in the automotive industry, in electronics, in civil and industrial construction [6]. Some examples of laminated composites with adhesive ties can be suggested: metal–metal joins, metal–composite material, composite material – composite material, honeycomb-type sandwich structures with metal surfaces, plastic or paper composites, and laminated composites reinforced with fibers. Examples of joining or adhesive ties are shown in Fig. 1 in the spirit of [7–9]. Fig.1 presents seven types of joining or adhesive ties: a) variable stiffness adhesive (beveled); b) variable stiffness adhesive 1 – stiff adhesive, 2 – less stiff adhesive; c) variable stiffness adhesive stiff adhesive; d) single taper; e) double taper; f) increase thickness; g) landed scarf joint. Metals used in laminated composites include aluminum, magnesium, steel, titanium and their alloys. The fibers used for reinforcement are carbon/graft, glass, polyethylene, iron. Adhesives are generally combinations of thermoplastic materials and elastomers [10, 11].

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For non-destructive evaluation (NDE) of the quality of adhesive joints there are several methods of investigation that include sonic, ultrasonic analysis, holography, radiography, thermography, acoustic emission and nuclear magnetic resonance. It is generally reported that a single method cannot solve a given problem. Each method has its limits. To date, there is no NDE procedure to detect the incipient weighting of the adhesive bond. Current NDE techniques detect the large-scale defects, delamination, but cannot identify the incipient weaknesses of the adhesive adhesive interface.

The use and architecture of new materials based on adhesives requires the characterization of their dynamic behavior [12]. Three stages of the fatigue processes are highlighted:

1. Perturbation of the crystal lattice leading to dislocations. The lattice perturbation energy is defined as the energy required to heat the material to melting point;

2. Initiation of microcracks in microregions of critical dislocation density;

3. Microcrack propagation. For this, the cohesion destroying energy is defined as the energy of melting.

Different epoxy systems were studied in [13] by using matrices in which were added the silica nanopowders. The sonication gradually breaks up the larger aggregates into smaller clusters leading to improved mechanical properties of the resulting nanostructures.

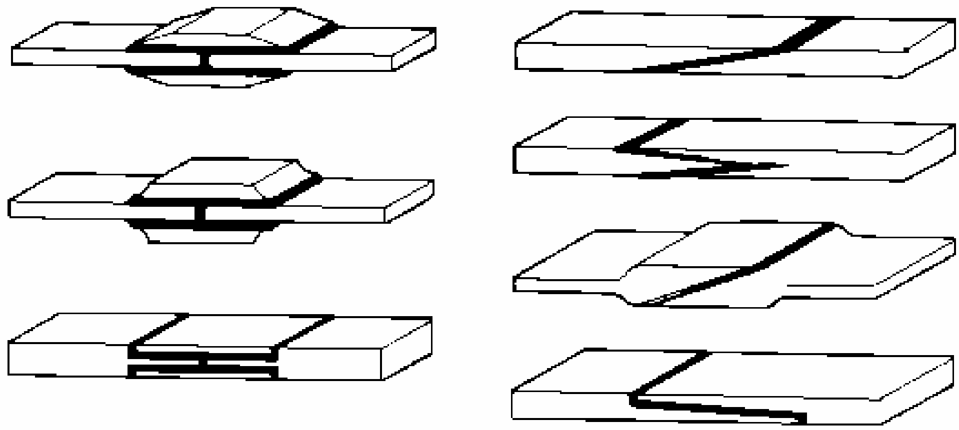

Fig. 1 – Examples of joining or adhesive ties: a) variable stiffness adhesive (beveled); b) variable stiffness adhesive, 1 – stiff adhesive, 2 – less stiff adhesive; c) variable stiffness adhesive stiff adhesive; d) single taper; e) double taper; f) increase thickness; g) landed scarf joint [7–9].

In the following, we shortly present the discrete and continuous approaches of fatigue. These laws are underlying to model he fatigue of laminated composites with adhesive bonds.

2. DISCRETE DAMAGE APPROACH

Starting hypothesis is that $U_c = U\rho_{cr}$ where U_c is the energy of a dislocation in a cluster of critical density ρ_{cr} and U is the mean energy of a dislocation of unit length

$$U = \frac{Gb^2}{2(1-\upsilon)}, \tag{1}$$

where $G = \dfrac{E}{2(1+\upsilon)}$, E is the Young modulus, υ is the Poisson ratio, and b is the Burger's vector. Comparison of this energy with the energy required to heat the material to melting point T_m yields

$$U_c = \int_{T_0}^{T_m} c_p \mathrm{d}T, \tag{2}$$

where T_0 is the temperature of the surroundings and c_p is the specific heat.

The critical dislocation density at which decohesion occurs is

$$\rho_{cr} = \frac{2(1+\upsilon)}{Gb^2} \int_{T_0}^{T_m} c_p \mathrm{d}T. \tag{3}$$

Ivanova [14] computed the Wohler curve for the critical failure stress as a function of number of cycles, and proposed an accelerated method for finding the fatigue limit. The thermal deformation or mechanical deformation leads to the loss of stability [15]. This model of decohesion is based on a critical parameter determined by the melting point. The critical strain ε_{cr} is compared with the strain due to the critical stress σ_{cr} under uniform triaxial tension with the thermal elongation during melting

$$\varepsilon_{cr} = \frac{1-2\upsilon}{E}\sigma_{cr} = \int_{T_0}^{T_m} \alpha \mathrm{d}T, \tag{4}$$

where α is thermal expansion coefficient. So, the critical value of the stress that causes the loss of stability and transition to a quasi-liquid state is given by

$$\sigma_{cr} = \frac{3EG}{3G-E} \int_{T_0}^{T_m} \alpha \mathrm{d}T. \tag{5}$$

Golaski [16] improved the Zakrzewski formula (5) by taking the lattice deformation into account through the introduction of dislocations. Hence ε_{cr} increases by $\varepsilon = \rho b^2$ and the critical stress becomes

$$\sigma_{cr} = \frac{3EG}{3G - E}\left(\int_{T_0}^{T_m} \alpha \mathrm{d}T - \rho b^2\right), \tag{6}$$

where b represents Burger's vector.

Further, Handzel [17] determined a law for the growth of dislocation density in a microvolume of metal subjected to cyclic stress as

$$\rho = \rho_0 (\exp \gamma\sigma_{\max} - \exp \gamma\sigma_{\min}) \ln N , \tag{7}$$

where ρ_0 is the initial dislocation density, $\sigma_{\max}$ is the maximal stress (in tension), $\sigma_{\min}$ is the minimal stress (in compression), N is the number of life cycles and γ is a material parameter that depends on the dislocation-activating stress (the slip stress).

Skelton [18] revealed that in the uniaxial case, for low cycle fatigue (at high temperature) the dissipated energy to failure (till the apparition of the microcrack) ΔW^{in} seems to be an intrinsic material constant (between $1J \times \mathrm{mm}^{-3}$ and $10J \times \mathrm{mm}^{-3}$). So, calculating the area of hysteresis loop in stress-strain curves, Δw^{in} meaning the energy per stabilized cycle, we can determine the number of life cycles N_f using the Skelon law

$$\Delta w^{in} \times N_f = \Delta W^{in} . \tag{8}$$

The fatigue degradation is associated with the evolution of micro-cracks in materials under cycling loads. The laws are formulated in terms of numbers of life cycles that can depend on the characteristics of a stabilized cycle (stress or strain amplitude, the medium, maxim or minim values of stress and strain). Three types of fatigues are distinguished [11-14]:

1. High cycle fatigue corresponding to a high number of life cycles ($N > 10^5$);

2. A fatigue characterized by a macroscopic elastic behavior, although, at microscopically level, stresses of some crystals can overpass the elastic range;

3. Low cycle fatigue corresponding to a small number of life cycles ($N < 10^5$) and characterized by important plastic deformations at macroscopically level.

Manson-Coffin law [3–5] was initially established for uniaxial loads. For the cycle fatigue, the basic assumption is that the deterioration on a cycle depends on the amplitude of inelastic (in special plastic) deformation $\Delta\varepsilon^{in}$ as a power law

$$\frac{\mathrm{d}D}{\mathrm{d}N} = f(\Delta\varepsilon^{in}) = \left(\frac{\Delta\varepsilon^{in}}{c_1}\right)^{\gamma_1}, \tag{9}$$

where D is degradation function which takes zero value for a non-damaged material and one for a degraded material, N is the number cycles, c_1 and γ_1 are material parameters that depend on temperature. In the case of the periodical loads, the cycles are stabilized and the integration of (9) with conditions

$$D(N=0)=0,\ D(N_f)=1, \tag{10}$$

leads to an expression for number of life cycles

$$N_f = \left(\frac{\Delta\varepsilon^{in}}{c_1}\right)^{-\gamma_1}. \tag{11}$$

For weak loads with small plastic deformations, the Mason's law can be corrected using the following degradation law for the stress amplitude $\Delta\sigma$

$$\frac{\mathrm{d}D}{\mathrm{d}N} = \left(\frac{\Delta\sigma}{c_2}\right)^{\gamma_2}, \tag{12}$$

where c_2 and γ_2 are material parameters that which depend on temperature, too.

By integration (12) with conditions

$$D(N=0)=0,\ D(N_f)=1, \tag{13}$$

we obtain the number of life cycles valuable for elastic deformations

$$N_f = \left(\frac{\Delta\sigma}{c_2}\right)^{-\gamma_2}. \tag{14}$$

If we couple the elastic and plastic deformations

$$\Delta\varepsilon = \Delta\varepsilon^{e} + \Delta\varepsilon^{in}, \tag{15}$$

with the linear elastic law

$$\Delta\sigma = E\Delta\varepsilon^{p}, \tag{16}$$

where E is Young modulus, a general law for low cycles fatigue as well as for high cycle fatigue can be obtained from (13) and (14)

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + \Delta\varepsilon^{in} = \frac{c_2}{E}\left(N_f\right)^{-\frac{1}{\gamma_2}} + c_1\left(N_f\right)^{-\frac{1}{\gamma_1}}, \tag{17}$$

with the coefficients E, c_1, γ_1, c_2 and γ_2 defined above.

We think it is better to use the so called *pentes universelles*

$$\Delta\varepsilon = \Delta\varepsilon^e + \Delta\varepsilon^{in} = 3.5\frac{\sigma_u}{E}(N_f)^{-0.12} + D_u^{0.6}(N_f)^{-0.6}, \tag{18}$$

where σ_u is ultimate stress of static rupture, and D_u is so called the ductility, that characterizes the reduction of the specimen's section.

The advantage of Manson-Coffin law consists in taking into account the degradation by low stress, as well as the degradation by inelastic (plastic) deformations, eventually with variable amplitude. The law contains a linear cumulation technique (the latest one presumes that the total degradation depends on the number of cycles effectuated through each mechanism in the case of complex loadings).

Another advantage of the Manson's law is that it can be simply adapted to multiaxial loads by substituting the unaxial stress or strain amplitudes with equivalent ones (von Mises, Tresca, etc.)

The disadvantages of Manson's law or other laws of this kind, for example Morrow or Berkovits' procedures [19] are used by the fact that they do not take into account stress relaxation at high plastic strains and they are not applicable for very high temperatures (bigger than those of creep degradation).

3. CONTINUOUS DAMAGE APPROACH

The continuous damage law was initially developed for the creep failure processes, the continuous damage approach has been extended to the fatigue processes or creep–fatigue interaction [20–32]. This approach is based on the concept of the damage internal variable D, which defines the actual strength of the material, taking into account the microscopic deteriorations induced by the cyclic loadings. Usually, $D = 0$ in the initial undamaged state, and $D = 1$ when the material is deteriorated. For pure creep processes Rabotnov proposed the following equation for deterioration [33]

$$\mathrm{d}D = \left(\frac{\sigma}{A}\right)^r (1-D)^{-k}\,\mathrm{d}t, \tag{19}$$

where A, r, k are parameters depending on temperature. Solving (19), the creep failure time for $D = 1$ is obtained

$$t_c = \frac{1}{k+1}\left(\frac{\sigma}{A}\right)^{-r}. \tag{20}$$

For pure fatigue damage processes, Chaboche proposed the equation [26, 27]

$$\mathrm{d}D = [1-(1-D)^{\beta+1}]^{\alpha(\sigma_M,\bar{\sigma})}\left(\frac{\sigma_M - \bar{\sigma}}{M(\bar{\sigma})(1-D)}\right)^{\beta}\mathrm{d}N\,, \tag{21}$$

with functions α and M defining the fatigue limit and the monotonic failure, σ_M is maximal stress, $\bar{\sigma}$ is mean stress and β, as the others coefficients contained in the expression of α and M, are material parameters depending on temperature. These coefficients are determined knowing the fatigue limit for different values of mean stress and Wohler curves.

The solution of (22) for $D=1$ gives the number of cycles to failure

$$N_F = \frac{1}{(\beta+1)[1-\alpha(\sigma_M - \bar{\sigma})]}\left(\frac{\sigma_M - \bar{\sigma}}{M(\bar{\sigma})}\right)^{-\beta}. \tag{22}$$

Following Chaboche [26] under low frequency cycling and high temperature, the creep and fatigue damage processes interact. As a result, the number of cycles to failure is considerable reduced.

In this case, the strain Range Partitioning Approach is proposed [27].

This means that a complex cycle is decomposed in four parts with four degradation relations independently determined. For example, one part is a cycle plastic in traction and the general equation of degradation is obtained through superposition

$$\mathrm{d}D = \frac{1}{N_{pp}}\frac{\Delta\varepsilon_{pp}}{\Delta\varepsilon_{in}} + \frac{1}{N_{cp}}\frac{\Delta\varepsilon_{cp}}{\Delta\varepsilon_{in}} + \frac{1}{N_{pc}}\frac{\Delta\varepsilon_{pc}}{\Delta\varepsilon_{in}} + \frac{1}{N_{cc}}\frac{\Delta\varepsilon_{cc}}{\Delta\varepsilon_{in}}\mathrm{d}N\,, \tag{23}$$

where N_{ij} and $\Delta\varepsilon_{ij}$ are the number of cycles till failure and strain amplitude for each kind of degradation, $\Delta\varepsilon_{in}$ is total strain amplitude. Here we use the notations as plastic in compression (pp); the second cycle is plastic in traction and on creep in compression (pc) and so on (cp), and (cc).

The Strain Range Partitioning Approach is proper to describe the intermediate range of frequency, including the differences between tension and compression damage but it is difficult to be applied in cases of temperature variations during the cycle or when the inelastic strain are small.

Another method proposed by Chaboche [26] is Continuous Damage Approach.

The damage equation is on the form

$$\mathrm{d}D = f(\sigma, D, T)\mathrm{d}t + g(\sigma_M, \bar{\sigma}, D, T)\mathrm{d}N\,, \tag{24}$$

where T is temperature, σ_M and $\overline{\sigma}$ are the maximum and mean stress during the cycle. The functions f and g represent the creep damage and fatigue damage, respectively. These functions are separately determined. This method includes HCF, non-linear cumulative effects but it is applicable for the materials having the same behavior in compression like in tension and when the cyclic hardening is not important.

On the other case, Continuous Damage Approach can be easier generalized to multiaxial cases than Strain Range Partitioning.

4. FATIGUE OF LAMINATED COMPOSITES WITH ADHESIVE BONDS

The laminated composites with adhesive bonds are modeled as elasto-visco-plastic materials, for which the classical elastic law is given by

$$\sigma_{ij} = \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)}\varepsilon_{ll}^{e}\delta_{ij} + \frac{E}{1+\upsilon}\varepsilon_{ij}^{e} \quad . \tag{25}$$

Decomposing the stress and strains tensors into deviatoric and volumetric part

$$\begin{cases} \sigma_{ij} = s_{ij} + \sigma^{h}\delta_{ij}, \\ \varepsilon_{ij}^{e} = e_{ij}^{e} + \varepsilon^{h}\delta_{ij}, \end{cases} \qquad \begin{cases} \sigma^{h} = tr\sigma / 3, \\ \varepsilon^{h} = tr\varepsilon / 3, \end{cases} \tag{26}$$

we obtain

$$\begin{cases} s_{ij} = \dfrac{E}{1+\upsilon}e_{ij}^{e}, \\ \sigma^{h} = \dfrac{E}{1-2\upsilon}\varepsilon^{h}. \end{cases} \tag{27}$$

Under cycling loadings, we obtain for the mean stress and strain

$$\begin{cases} \sigma_{mij} = s_{mij} + \sigma_{m}^{h}\delta_{ij}, \\ \varepsilon_{mij}^{e} = e_{mij}^{e} + \varepsilon_{m}^{h}\delta_{ij}, \end{cases} \begin{cases} s_{mij} = \dfrac{E}{1+\upsilon}e_{mij}^{e}, \\ \sigma_{m}^{h} = \dfrac{E}{1-2\upsilon}e_{m}^{h}. \end{cases} \tag{28}$$

For description the fatigue criterion the following notations are used

$$\begin{cases} \overline{\sigma}_{ij} = \sigma_{ij} - \sigma_{mij}, \\ \overline{\varepsilon}_{ij} = \varepsilon_{ij}^{e} - \varepsilon_{mij}. \end{cases} \tag{29}$$

$$\begin{cases} \bar{\sigma}_{ij} = \bar{s}_{ij} + \bar{\sigma}^h \delta_{ij}, \\ \bar{\varepsilon}^e_{ij} = \bar{e}^e_{ij} - \bar{\varepsilon}^h \delta_{ij}. \end{cases} \tag{30}$$

Writing

$$\begin{cases} \bar{s}_{ij} = s_{ij} - \sigma_{m\,ij} + \sigma^h_m \delta_{ij}, \\ \quad \bar{\sigma}^h = \sigma^h - \sigma^h_m, \\ \bar{\varepsilon}^e_{ij} = e^e_{ij} - \varepsilon_{m\,ij} + \varepsilon^h_m \delta_{ij}, \\ \quad \bar{\varepsilon}^h = \varepsilon^h - \varepsilon^h_m, \end{cases} \tag{31}$$

we obtain

$$\begin{cases} \Delta\sigma_{m\,ij} = 0, \\ \Delta\sigma^h_m = 0, \\ \Delta\varepsilon_{m\,ij} = 0, \\ \Delta\varepsilon^h_m = 0, \end{cases} \quad \begin{cases} \Delta\bar{s}_{ij} = \Delta s_{ij}, \\ \Delta\bar{\sigma}^h = \Delta\sigma^h, \\ \Delta\bar{e}_{ij} = \Delta e_{ij}, \\ \Delta\bar{\varepsilon}^h = \Delta\varepsilon^h. \end{cases} \tag{32}$$

Park and Nelson [34] suggest that the strain energy density under cyclic loading W is the sum of a variable term ΔW and a static (mean) one W^h_m that is associated with the mean hydrostatic stress

$$W = \Delta W + W^h_m, \tag{33}$$

$$W_m = \frac{3}{2}\sigma^h_m \varepsilon^h_m = \frac{3}{2}\frac{1-2\upsilon}{E}\sigma^{h\,2}_m. \tag{34}$$

The variable term is decomposed in deviatoric (elastic plus plastic) part and volumetric part.

$$\Delta W = \Delta W^e + \Delta W^p = \Delta W^{ed} + \Delta W^h + \Delta W^{pd}, \tag{35}$$

where ΔW^{pd} denotes the amount of plastic strain energy dissipated irrecoverably during a cycle

$$\Delta W^{pd} = \int_{cycle} s_{ij} \mathrm{d}e^p_{ij} = \int_{cycle} \sigma_{ij} \mathrm{d}\varepsilon^p_{ij}. \tag{36}$$

The term ΔW^{ed} is the elastic variable part of the energy given by

$$\Delta W^{ed} = \int\limits_{cycle} \left\langle \bar{s}_{ij} \mathrm{d}\bar{e}_{ij}^{e} \right\rangle = \frac{\Delta \bar{s}_{ij}}{2} \frac{\Delta \bar{e}_{ij}}{2} = \frac{\Delta s_{ij}}{2} \frac{\Delta e_{ij}}{2} = \frac{1+\upsilon}{4E} \Delta s_{ij}^{2} = \\ = \frac{2(1+\upsilon)}{3E} s_{eq}^{2}, \tag{37}$$

where

$$s_{eq} = \sqrt{\frac{3}{8} \Delta s_{ij}^{2}}\ , \tag{38}$$

is the equivalent deviatoric stress amplitude.

The term ΔW^{h} represents the volumetric elastic variable part of the energy given by

$$\Delta W^{h} = \int\limits_{cycle} \left\langle 3\bar{\sigma}^{h} \mathrm{d}\bar{\varepsilon}^{h} \right\rangle = 3 \frac{\Delta \bar{\sigma}^{h}}{2} \frac{\Delta \bar{\varepsilon}^{h}}{2} = 3 \frac{\Delta \sigma^{h}}{2} \frac{\Delta \varepsilon^{h}}{2} = \frac{3}{4} \frac{1-2\upsilon}{4E} \Delta \sigma^{h2}. \tag{39}$$

The term ΔW^{dp} tends to be bigger than the others, so it can be adopted as a damage parameter. Empirically, (39) can be related to the fatigue life

$$\Delta W^{dp} = AN^{\alpha}\ , \tag{40}$$

where A and α are the material parameters computed from uniaxial curves fatigue data.

On the contrary, the elastic distortion strain energy has a bigger cyclically variation. Empirically too, the number of life cycles is correlated with

$$\Delta W^{ed} = BN^{\beta}, \tag{41}$$

where the parameters B and β are determined from fully reversed uniaxial fatigue data.

In order to apply over the entire range of cycles, two energy terms are combined into the total strain energy parameter

$$\Delta W^{dp} + \Delta W^{ed} = AN^{\alpha} + BN^{\beta}, \tag{42}$$

where the constants A, B, α, β from above.

Park and Nelson [34] interpreted the term ΔW^{h} as the energy at the stress state. More precisely, the stress state may be characterized by the ratio

$$\frac{\Delta W^{h}}{\Delta W^{ed}} = \frac{\frac{3}{4} \frac{1-2\upsilon}{E} \Delta \sigma^{h2}}{\frac{2(1+\upsilon)}{3E} s_{eq}^{2}} \approx \frac{\Delta \sigma^{h}}{s_{eq}} = \frac{2}{3} \frac{(\sigma_1 + \sigma_2 + \sigma_3) a}{s_{eq}}, \tag{43}$$

where $\sigma_1, \sigma_2, \sigma_3$ are principal values of amplitude stress.

They introduced a term which give a unique value for each cyclic stress state

$$TF_s = \frac{(\sigma_1 + \sigma_2 + \sigma_3)a}{s_{eq}} = \frac{3}{2}\frac{\Delta W^h}{\Delta W^{ed}} . \tag{44}$$

This term is zero for torsion, takes the value 1 for tension-compression and takes the value 2 for in plane equiaxial stress state. Using this term, the effective plastic work per cycle is defined as

$$\Delta W^{p^*} = 2^{k_1(TFs-1)} \Delta W^p , \tag{45}$$

where k_1 is a constant that can be determined from two sets of test data with different stress states.

The fatigue law is then corrected by using this effective plastic work instead of ΔW^p.

The term W_m^h can be regarded as a measure of mean stress. The mean stress effect can be characterized by the ratio

$$\frac{\Delta W_m^h}{\Delta W^{ed}} = \frac{\frac{3}{2}\frac{1-2\upsilon}{E}\sigma_m^{h^2}}{\frac{2(1+\upsilon)}{3E}s_{eq}^2} \approx \frac{\sigma_m^h}{s_{eq}} = \frac{1}{3}\frac{(\sigma_1 + \sigma_2 + \sigma_3)m}{s_{eq}} , \tag{46}$$

where $\sigma_1, \sigma_2, \sigma_3$ are principal values of mean stress.

The triaxiality factor for mean stress

$$TF_m = \frac{(\sigma_1 + \sigma_2 + \sigma_3)m}{s_{eq}} = \frac{3}{2}\frac{\Delta W_m^h}{\Delta W^{ed}} , \tag{47}$$

can be used for defining the effective elastic distorsion strain energy parameter

$$\Delta W^{e^*} = 2^{k_2 TF_m} \Delta W^e , \tag{48}$$

where the constant k_2 can be determined from fully reversed $(TF_m = 0)$ and zero to maximum $(TF_m = 1)$ fatigue data.

The fatigue law can be corrected again by using this effective elastic strain energy instead of ΔW^e.

$$\Delta W^{e^*} + \Delta W^{p^*} = AN^{\alpha} + BN^{\beta} , \tag{49}$$

where N is number of life cycles, and parameters A and α are uniaxial curves $\varepsilon^p - N$ fatigue data and B and β are determined from fully reversed uniaxial fatigue data.

5. EXAMPLE

For exemplification of the fatigue law (49), we analyze by using ANSYS, the 1-D displacement of a laminated composite beam with adhesive bonds under applied loading 10 000 N and boundary conditions: the bar is embedded at the left end and free at the right end. The geometry of the bar is presented in Fig. 2. The inclination of the adhesive layer is 45^0. The material properties of the adherents are the Young modulus $E = 70$ GPa, Posson ratio $\upsilon = 0.3$, and the material properties of the adhesive are $E = 4.82$ GPa and $\upsilon = 0.4$.

We divided the bar into 100 linear rectangular elements 1D (mm) and 101 nodes 10 finite elements and 11 nodes. Each element has two end-points where it either joins another element or interacts with constraints and the load.

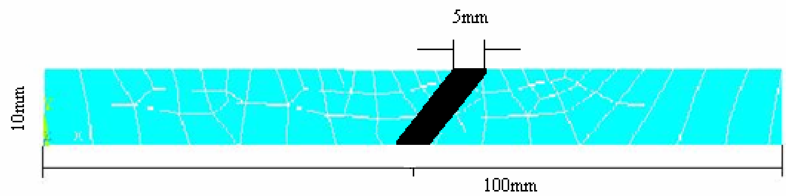

Fig. 2 – Laminated composite beam with adhesive bonds.

The longitudinal components of stress and strains tensors are computed and presented in Figs. 3 and 4.

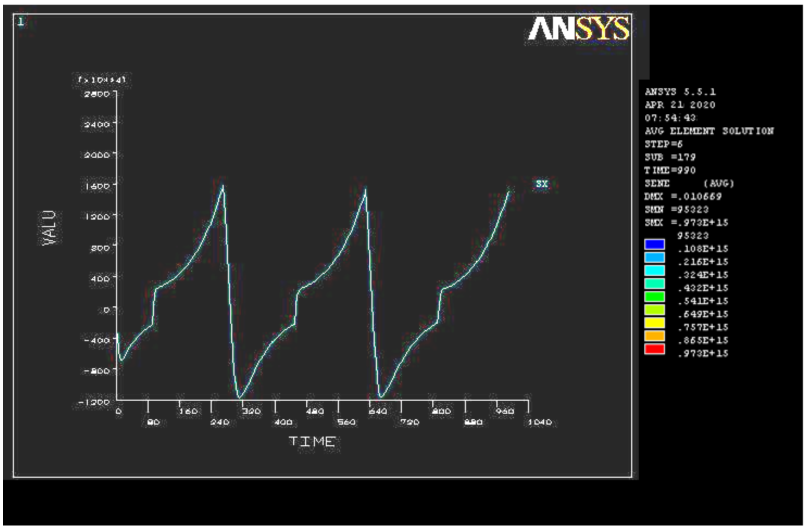

Fig. 3 – Stress field variation in the beam.

The dissipated energy on a stabilized cycle is computed for both adhesive and adherent. Their behavior is somewhere in between perfectly elastic and perfectly inelastic. The variation of the stress with respect to total strain in the beam is presented in Fig. 5.

The distribution of the inelastic dissipated energy can be seen in Fig. 6. As we expected, there is an important loss of inelastic energy in the bonding zone. That means that this zone would be that fails.

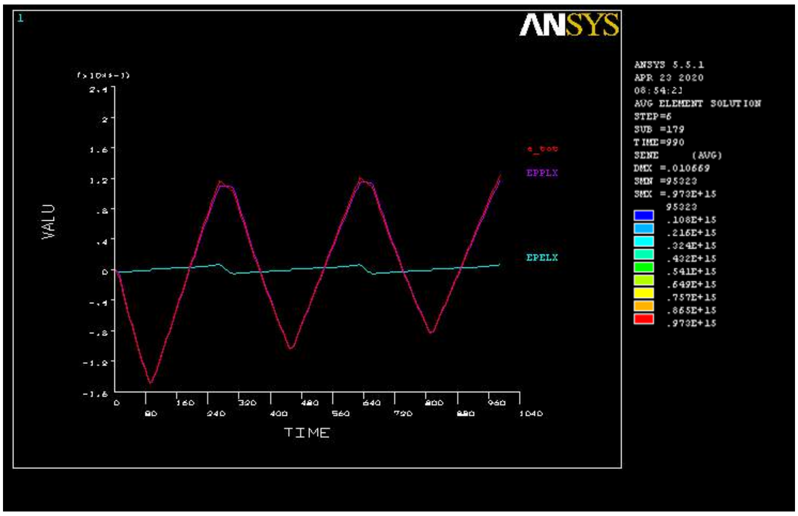

Fig. 4– Strains field in the beam.

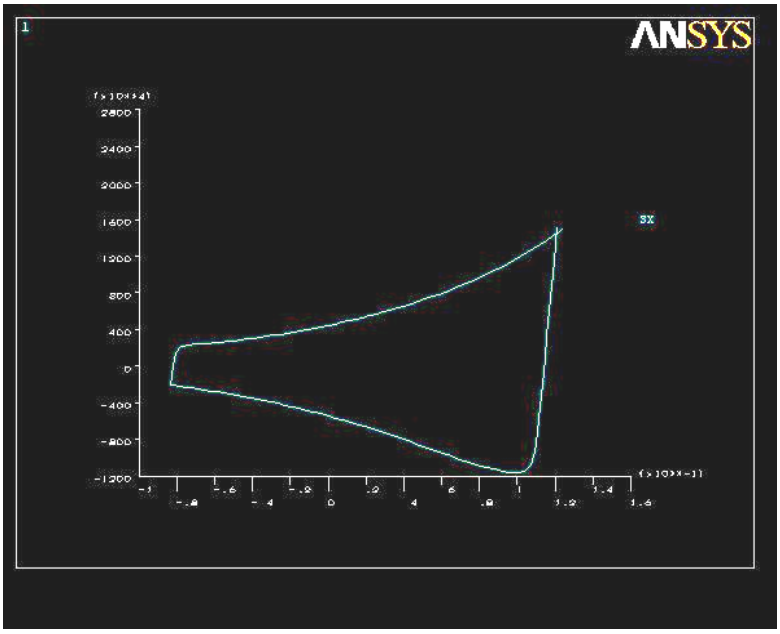

Fig. 5 – Stress versus total strain in the beam.

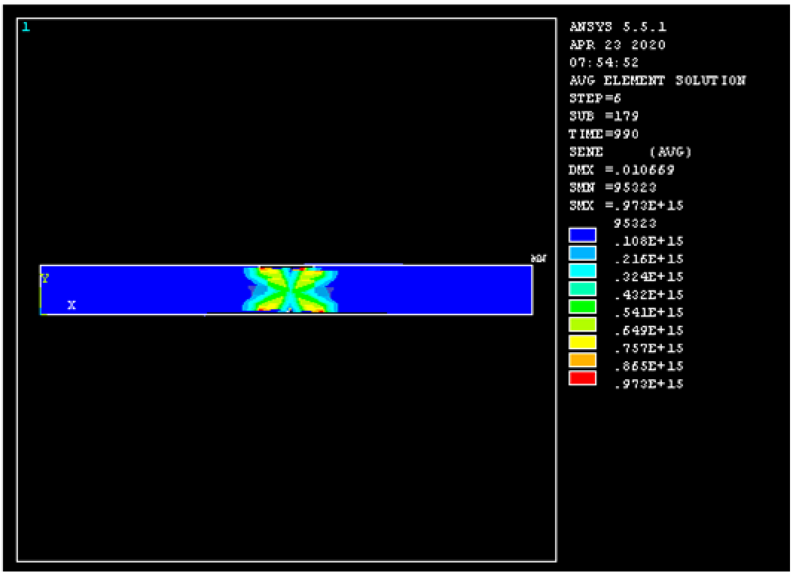

Fig. 6 – Distribution of dissipated inelastic energy in the beam.

5. CONCLUSIONS

Composite structures with adhesive joins are intensively used in engineering mechanics. The fatigue of laminated composites with adhesive bonds is studied in this paper. Discrete and continuous approaches of fatigue are underlying to modeling the fatigue of laminated composites with adhesive bonds. Pre-existing or material-induced microscopic defects during manufacturing play an important role in generating macro-defects (delamination, cracks, voids) in both adhesive and adherent. The propagation of defects and the way of breaking (adhesive or adherent) depend on a series of parameters including the type of material, the design of the bond, the environmental conditions, the type of surface treatment, the quality of the joining process, etc.

Under low frequency cycling and high temperature, the creep and fatigue damage processes interact. As a result, the number of cycles to failure is considerable reduced.

The distribution of the inelastic dissipated energy shows that there is an important loss of inelastic energy in the bonding zone. That means that the bonding zone would be that fails.

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