STRESS UNIFORMIZATION USING FUNCTIONALLY GRADED MATERIALS

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Abstract. A particular geometric domain with specified boundary conditions which remain constant is considered. The elastic properties of the solid isotropic material, as Young's modulus of elasticity E and Poisson's ratio ν, are variable pointwise. From practical point of view, one may encounter materials with similar values of the transversal contraction coefficients and therefore we assume that ν is constant. The problem to be analyzed is the optimization of the distribution of the material properties as to minimize and/or get a uniform distribution of the state of stress in each domain. The used strategy of optimization is a heuristic method, that is the fully stressed design (FSD) technique. Two simple applications, for pin jointed bars and a circular stress raiser, are presented as to validate the proposed calculus methodology.

Keywords: Elastic analysis, functionally graded material (FGM), full stressed design (FSD), iterative technique.

1. INTRODUCTION

The level of loading is usually quantified through the calculation of an equivalent stress. The distribution of the stresses in a structure usually shows their maximum values in a limited number of points. If the topology of the structure is not modified the variation of the stresses can be modified by changing the material properties according to a constitutive law as to obtain the same maximum loading in a larger domain, preferably in the whole structure. The simplest example in this sense is a bar/plate with uniform cross section made from an isotropic and homogeneous material loaded in traction. The bending and/or twisting loadings or the presence of stress raisers are leading to non-uniform distributions of stresses. Their maximum values can be reduced by controlling the distribution of material properties.

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For an isotropic material (defined through E and ν), obtained through the combination of at least two components, one “soft” and one “rigid”, may result important variations of the material’s constants in a large domain of the order of units or even orders of magnitude.

Functionally graded materials (FGMs) represent a class of novel materials in which compositions/constituents and/or microstructures gradually change along single or multiple spatial directions, resulting in a gradual change in properties and functions which can be tailored for enhanced performance. These materials can be obtained through additive manufacturing [1, 2] and are more often used in industrial applications [3, 4]. More recently, strategies for complex optimization, both of topology and of design, were proposed as to fulfill multiple criteria [4, 5].

The issue of optimization, although proposed more than 100 years ago, evolved considerably. There is an extremely large number of information and publications in this domain, but in the recent period the used approaches and the mathematical apparatus have diversified considerably [6–8]. The classical analytical analyses were extended towards the numerical ones and today most of the practical applications are based on the finite element method (FEM). The dedicated software packages do not generally contain explicit implementation for modeling functionally graded materials (FGMs), as most of them use isotropic or anisotropic homogeneous materials. This drawback was initially overcome by implementing finite elements with a nonhomogeneous material (FGM) [9, 10], but a sufficiently good precision can be obtained by using refined meshes for which each element is considered an isotropic material [9].

A large category of academic applications dedicated to the optimization issues were developed by using truss structures [6, 11]. These can be dimensioned as being fully stressed, that is in all the bars made from the same material the optimum design is obtained when same stress is obtained, and the *fully stressed design* (FSD) concept is used. For statically determined structures the problem is rather simple, but for statically indetermined structures in general a solution does not exist, and for its existence certain restrictions and supplementary corrections [12, 13] must be imposed, as considering a unique loading case. However, in practice, such structures are optimized for several loading cases and dedicated software as [14] is used, but the solution is obeying the condition of full stress only for a limited number of structural components.

For simple continuous structures loaded symmetrically (spheres, thick-walled tubes, rotating disks) were established [15, 16] conditions for which in the whole geometrical domain reaches certain stresses or even the equivalent stresses are the same, that is the FSD design is used. These conditions can be fulfilled if the material has isotropic properties but is nonhomogeneous, that is FGM. For solving such relatively complex problems usually analytical methods are used. Numerical methods with complex approaches [17] can be used only for validation or to obtain solutions for given laws of variation for the properties of the FGM.

It is expected that a structure loaded in the condition of full stress will lead to a condition of minimum weight design [18], but this conclusion is not always true.

If for optimization and full stress condition is used a FGM, as Young's modulus is variable it is probable that the failure criteria and the density of the material will be variable, which leads this approach to a complicated problem. The method may become efficient from practical point of view if instead of designing under the fully stressed concept is used another parameter as the *structural efficiency percentage* (SEP) [19] which is equivalent to attaining the limit state of stress in all points of a structure simultaneously.

This work is dedicated to defining a relatively new category of optimization problems. Starting from the fact that additive printing allows the obtaining of FGMs it is desired to equalize some well-established variables. In our case we consider only the equivalent stresses. Using the hypothesis that the failure limit of a FGM does not essentially depend on the current (local) values of the materials' constants it can be argued that as compared to the full stress approach used for bars where is established the optimized cross-section, we can determine the variable value of the modulus of elasticity by keeping the cross-section constant. The results can be immediately transferred to the continuous structure if the FEM method is used, and it is accepted that the variation of the elastic properties is done from element to element. From practical point of view the optimization problem defined in this paper can lead to a quasi-continuous solution for the variation of the FGM material properties under the optimization criterion based on the largely used heuristic method of optimization – fully stressed design.

The paper briefly presents the optimum design problem formulation in the second section and established the optimization strategy in the third section. Then, in Section 4, the optimization algorithm is shortly overviewed by pointing out the specific issues on convergence and algorithm validation. For verification, in Section 5 two applications are given as examples: a 2D system of pin jointed bars under three loading cases and a thin plate with a circular stress raiser loaded biaxially. Comments on the material modulus of elasticity values and distributions are done and for each application conclusions are drawn.

2. OPTIMUM DESIGN PROBLEM FORMULATION

For a finite element model, meshed with n finite elements were considered as variables the moduli of elasticity of all the elements E_j, $j = 1 \ldots n$, like the approach used for topological optimization. It is defined

$$\boldsymbol{X} = \left[E_1 \quad E_2 \quad \cdots \quad E_n \right], \tag{1}$$

as the vector of the unknowns or the design variables. This definition and approach for the design variables was first used in [20] as to simulate adaptive bone mineralization by varying the Young's modulus according to a calculated stress distribution. For each finite element can be associated an average value of an equivalent stress $\sigma_{eqv,j}$. A function $F : R^n \to R$ can be defined through the extreme global values of the equivalent stresses or other global parameters. For example, $\sigma_{eqv,\max} = \max_j \left(\sigma_{eqv,j}\right)$ and $\sigma_{eqv,\min} = \min_j \left(\sigma_{eqv,j}\right)$. The following function can be defined

$$F\left(\boldsymbol{X}\right) = \sigma_{eqv,\max} - \sigma_{eqv,\min}, \qquad (2,a)$$

through two global values. If the global values are equal, then $F\left(\boldsymbol{X}\right) = 0$ and the equivalent stresses are constant and uniformly distributed over the whole domain to be considered for FSD. In most situations the minimum equivalent stress does not have a significant practical importance and we can define

$$F\left(\boldsymbol{X}\right) = \sigma_{eqv,\max}. \qquad (2,b)$$

Therefore, the problem of optimization *minmax* becomes a continuous variable optimization criterion

$$\textit{Minimize} \quad F\left(\boldsymbol{X}\right), \text{ Objective function} \qquad (3)$$

subjected only to side constraints

$$E_{\min} \le E_j \le E_{\max}; \quad j = 1 \ldots n, \qquad (4)$$

where $E_{\min}$ and $E_{\max}$ define the domain of acceptable values for the design variables.

Based on the observation that the equivalent stresses (or other similar variables) are diminished when the material is “softer” an algorithm which “adapts” the stiffness (longitudinal modulus of elasticity) of each finite element was proposed, so that, for a given calculus model, the equivalent stresses will become equal. In fact, this procedure is nothing else than an adapted fully stressed design.

As a simplification, the coefficient of transversal contraction was considered constant, but its influence is reduced compared to the one given by the modulus of elasticity [16, 17].

3. OPTIMIZATION STRATEGY

The relation of iterative “correction” of the modulus of elasticity was implemented by using the heuristic method (fully stressed design). It is considered that all the elements *j* are made from the same material with the modulus of elasticity E_0, and for each iteration (*i*), in each element the modulus of elasticity (as a design variable) is updated by using the law

$$E_j^{(i)} = E_j^{(i-1)}\left[1+\alpha\left(1-\frac{\sigma_{eqv,j}^{(i-1)}}{\sigma_{eqv,\max}^{(i-1)}}\right)\right], \qquad (5,a)$$

in which:

- α is a scalar value which defines the "degree of modification" of the modulus of elasticity and which in this work was maintained constant throughout the iterations;

- $\sigma_{eqv,j}^{(i-1)}$ is the variable, which is to become uniform, as with notation being the equivalent stress from element *j* at the iteration *i*-1;

- $\sigma_{eqv,\max}^{(i-1)}$ is the maximum equivalent stress for all the elements at iteration *i*-1.

Sometimes it is more convenient for $\sigma_{eqv,\max}^{(i-1)}$ to be replaced by $\sigma_{eqv,\mathrm{med}}^{(i-1)}$ and the relation of correction is transformed into

$$E_j^{(i)} = E_j^{(i-1)}\left[1+\alpha\left(1-\frac{\sigma_{eqv,j}^{(i-1)}}{\sigma_{eqv,\mathrm{med}}^{(i-1)}}\right)\right], \qquad (5,b)$$

where

- $\sigma_{eqv,\mathrm{med}}^{(i-1)} = \dfrac{\sigma_{eqv,\min}^{(i-1)} + \sigma_{eqv,\max}^{(i-1)}}{2}$ is the median equivalent stress at iteration *i*-1.

Relation (5,a) makes a correction as increasing continuously the modulus of elasticity. Therefore, the possible decrease of the modulus below zero is avoided.

It is to be again underlined that instead of the equivalent stress one may consider another similar parameter, as a component of the stress tensor, the specific strain energy or even parameters which can be derived from stresses as the structural efficiency percentage, as it was defined in [19].

In the present implementation the iterative correction of the design variables is done only by considering the current information obtained from the last iteration. An improved variant can be obtained if the correction is done by considering the last steps of iteration, but this a subject for a future work.

Convergence criterion

For stopping the calculus by using this algorithm the infinity norm of relative variation of the modulus of elasticity was used for the last two iterations,

$$\max_j\left(\frac{\left|E_j^{(i)} - E_j^{(i-1)}\right|}{E_j^{(i)}}\right) \le \varepsilon_E\,, \qquad (6,a)$$

where, depending on the application, one can chose $\varepsilon_E = 10^{-3}$ - 10^{-6}.

Another option for stopping the calculation can be established based on the extreme stresses obtained at the current iteration, that is

$$\frac{\sigma_{eqv,\max}^{(i)} - \sigma_{eqv,\min}^{(i)}}{\sigma_{eqv,\max}^{(i)}} \le \varepsilon_\sigma \,, \tag{6,b}$$

for which, depending on the problem, ε_σ can be chosen in between 10^{-1} - 10^{-3}. This criterion is efficient for problems of FSD type.

Correction of the minimum limit

In the process of iteration, it is possible that for at least an element the modulus of elasticity would drop below the initial value, that is E_0, especially if the algorithm is based on the value of the median stress. To avoid obtaining negative values of the modulus of elasticity and for obtaining a unique solution for the distribution of the moduli of elasticity it can be chosen (arbitrarily) that the minimum value of the modulus will be exactly E_0. Thus, at each iteration (or at the end of the calculations as to reduce the calculus effort) the obtained values $E_j^{(i)}$ can be uniformly scaled because the solution of the problem without restrictions is not unique [12, 15], and it depends on a constant which can be eventually included as a requirement for defining the problem [6, 13], and therefore

$$E_j^{(i)} = \frac{E_0}{\min\limits_j \left(E_j^{(i)}\right)} E_j^{(i)} \text{ ; for } j = 1 \ldots n \,. \tag{7}$$

Algorithm verification

The algorithm was implemented in ANSYS APDL for various models. Additionally, functions were written in the software Octave for the analytical implementation of the relations presented previously as to compare the analytical results with the numerical ones.

4. OPTIMIZATION ALGORITHM

The proposed method is now described in detail.

• **Step 1: Set all algorithm constants**

Which means to establish the maximum permitted iteration N_{iter} if no convergence is assured; the permissible errors ε_E and/or ε_σ; the step size α; the objective function $F(\boldsymbol{X})$; the optimization strategy, i.e., the Eq. (5,a) or Eq. (5,b); the range of design variables $E_{\min}$ and $E_{\max}$.

• **Step 2: Implement a particular problem**

After completely defining a problem, the geometry, loads and supports, finite element types are considered to build the finite element model. Every finite element j has attached a material identification number with elastic constants and the Young's modulus E_j represents a design variable.

• **Step 3: Initialization**

For the first step i = 1, set $E_j^{(i)} = E_0$; $j = 1 \ldots n$. If Eq. (5,a) is used for updating the design variables, then $E_0 = E_{\min}$ is a good choice.

• **Step 4: Solve the problem for current configuration**

Solving for one iteration for a static solution for a single load case.

• **Step 5: Evaluate the objective function**

At the current iteration (i), for all element first are computed the stress components, then $\sigma_{eqv,j}^{(i)}$ and $\sigma_{eqv,\max}^{(i)} = \max_j\left(\sigma_{eqv,j}^{(i)}\right)$ and $\sigma_{eqv,\min}^{(i)} = \min_j\left(\sigma_{eqv,j}^{(i)}\right)$.

• **Step 6: Update the design variables**

Using (5,a) or (5,b) the design variables are updated.

• **Step 7: Check constraints**

If the relations (4) are not satisfied, then simply limit the design variables to the feasible design field:

if $E_j^{(i)} < E_{\min}$ then $E_j^{(i)} = E_{\min}$,

if $E_j^{(i)} > E_{\max}$ then $E_j^{(i)} = E_{\max}$.

• **Step 8: Design variables normalization (optional)**

Because generally the solution is not unique, it may be useful to control a particular value in the design variables. For example, if the minimum value of the design variables is enforced to be $E_{\min}$, Eq. (7) may be used for the uniform scaling of the design variables. This step may be also done only once at the end, if necessary.

• **Step 9: Check for convergence criterions and termination conditions**

If the convergence criterion (6,a) and/or (6,b) are satisfied, then stop, otherwise set $i = i+1$ and return to Step 4. Supplementary, if the convergence criterions are not satisfied, then the termination condition is assured when $i = N_{\text{iter}}$.

5. ALGORITHM VERIFICATION

5.1. APPLICATION 1: 2D PIN JOINTED BARS

Consider the geometry of the Navier's problem [21**]** for the 7-bar pin jointed framework shown in Fig. 1. The common joint 8 is subjected to a force of components F_x and F_y. Determine the axial stresses and the displacement of joint 8 in the condition of stress uniformization for adequate load conditions.

Denoting N_j the forces applied in each bar to the end nodes, the equilibrium equations of joint 8 can be written

$$F_x = \sum_{i=1}^{7} N_j \cos\theta_j ; \quad F_y = \sum_{i=1}^{7} N_j \sin\theta_j \ . \tag{8}$$

The force-displacement relations [21] are

$$N_j = \frac{E_j A}{a}\left(u_8 \cos\theta_j + v_8 \sin\theta_j\right)\sin\theta_j ; \quad j = 1\ldots 7 \ . \tag{9}$$

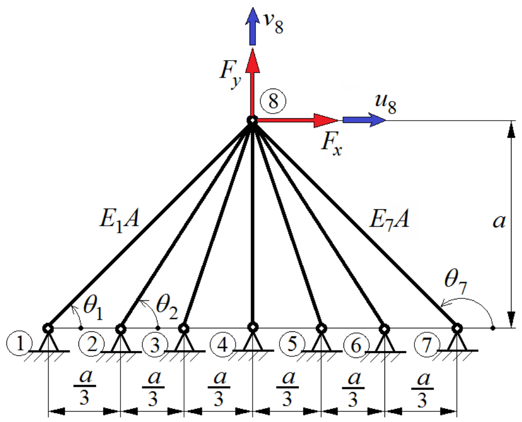


Fig. 1 – Application 1: a symmetric system with seven pin jointed bars.

If it is judiciously enforced $\sigma_j = \frac{N_j}{A} = \pm\sigma_0 = \text{const.}$, $j = 1\ldots 7$, using (8) it results

$$F_x = A\sigma_0 \sum_{j=1}^{7} \pm\cos\theta_j ; \quad F_y = A\sigma_0 \sum_{j=1}^{7} \pm\sin\theta_j \ . \tag{10}$$

where the signs are according to the particular choice of the stresses.

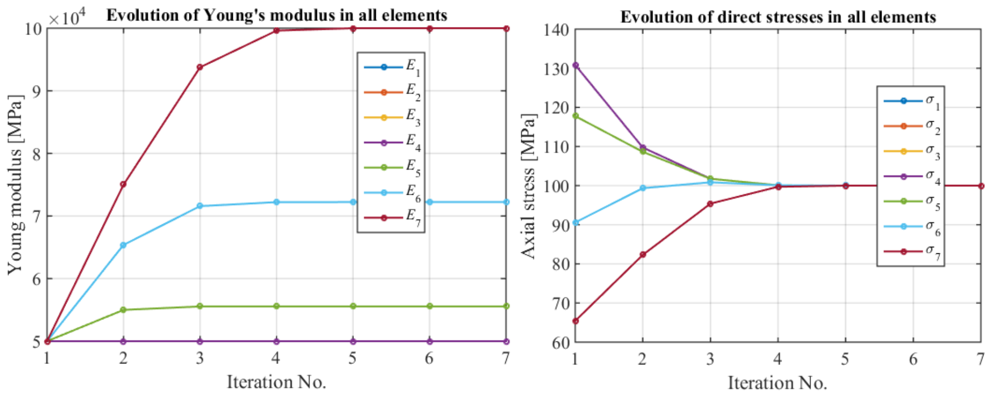
Loading case 1 (LC1)

For example, lets define loading case LC1: $\sigma_j = \frac{N_j}{A} = \sigma_0 = \text{const.}$, $j = 1\ldots 7$. Due to the symmetry $\sum_{j=1}^{7} \cos\theta_j = 0$ and u_8 = 0. A solution with $\sigma_j = \sigma_0$ is possible only if F_x = 0. For $\sigma_j = \sigma_0 = 100$ MPa and A = 100 mm^2 it results for vertical force F_y = 59.757 kN. Using (9) and u_8 = 0, it results

$$E_j = \sigma_0 \frac{a}{v_8 \sin^2 \theta_j}\ ;\quad j = 1 \ldots 7\,. \tag{11}$$

If we consider a = 1000 mm and v_8 = 2 mm, we obtain the solutions by neglecting the constraints (4). These are: $E_1 = E_7$ = 100 GPa; $E_2 = E_6$ = 72.22 GPa; $E_3 = E_5$ = 55.56 GPa and E_4 = 50 GPa.

Using the proposed algorithm and Eq. (5,a) it was verified that the best convergence ratio is given for α = 1.0. The obtained results by considering the convergence criteria (6,a) with $\varepsilon_E = 10^{-6}$ are presented in Fig. 2.

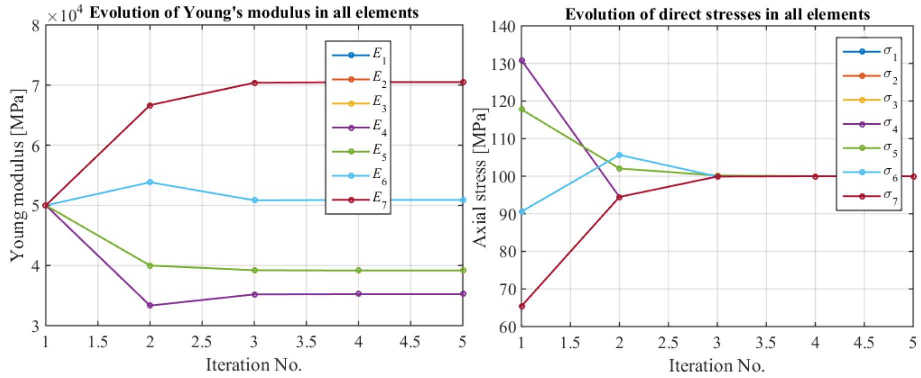


(a) Design variables convergence (b) Stress convergence

Fig. 2 – Numerical results for Application 1, LC1, considering α = 1.0, Eq. (5,a) and $\varepsilon_E = 10^{-6}$.

For α = 0.5 the convergence is obtained in 22 iterations; for α = 0.2 the convergence is obtained in 58 iterations and if α = 2 or larger, due to the oscillations in convergence, the solutions were not obtained for the maximum number of permitted iterations set as N_{iter} = 60.

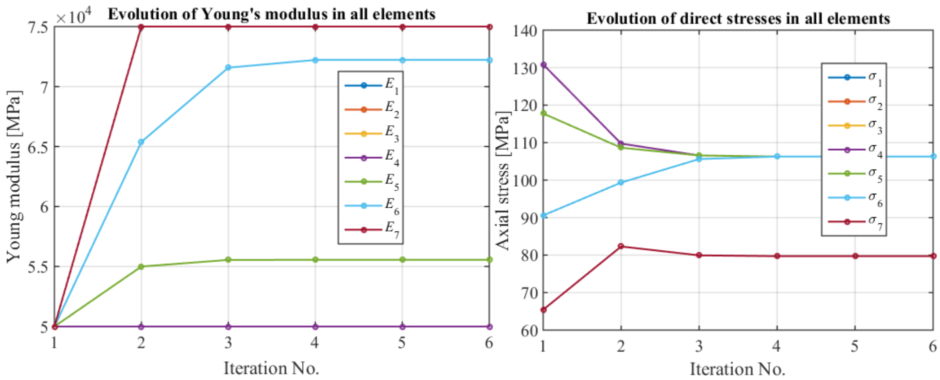
Using the proposed algorithm, but Eq. (5,b) for α = 1.0, without normalization of design variables, the obtained results are presented in Fig. 3. This is also a correct solution but for v_8 = 2.8882 mm.



(a) Design variables convergence (b) Stress convergence

Fig. 3 – Numerical results for Application 1, LC1, considering α = 1.0, Eq. (5,b) and $\varepsilon_E = 10^{-6}$.

If restrictions are considered, that is: $E_0 = E_{min}$ = 50 GPa, and $E_{max} = 1.5 \cdot E_{min}$ the resulting convergence curves are given in Fig. 4.



(a) Design variables convergence (b) Stress convergence

Fig. 4 – Numerical results for Application 1, LC1, considering α = 1.0, Eq. (5,b), the normalization given by Eq. (7), restriction $50\,\text{GPa} \le E_j \le 75\,\text{GPa}; \quad j = 1 \ldots 7$ and $\varepsilon_E = 10^{-6}$.

Using the mentioned restrictions, it results: $E_1 = E_7$ = 75 GPa; $E_2 = E_6$ = 72.22 GPa; $E_3 = E_5$ = 55.56 GPa and E_4 = 50 GPa and for nodal displacement v_8 = 2.1258 mm. Thus, as compared to the case without restrictions, only the values of the elasticity moduli in bars 1 and 7 are reduced. The distribution of the stresses results: $\sigma_1 = \sigma_7$ = 79.716 MPa; $\sigma_2 = \sigma_6 = \sigma_3 = \sigma_5$ = 106.29 MPa and σ_4 = 50 MPa.

Loading case 2 (LC2)

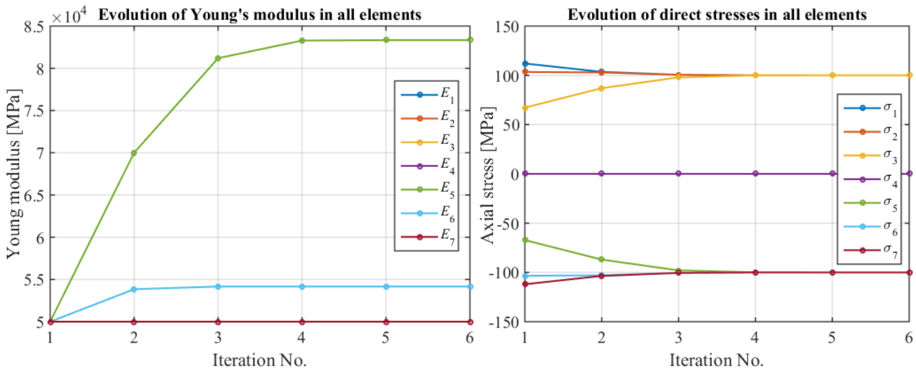
If we define LC2 for F_y = 0, i.e., an anti-symmetric loading case, and due to anti-symmetry, it results v_8 = 0, stresses are anti-symmetric and thus σ_4 = 0. Taking

$|\sigma_i| = \sigma_0 = 100$ MPa (σ_1 = σ_2 = σ_3 = 100 MPa; σ_5 = σ_6 = σ_7 = -100 MPa) for the remaining trusses and the same geometrical data (A = 100 mm^2 and a = 1000 mm), from (10) results F_x = 31.5607 kN. Then from (9) we obtain

$$E_j = \pm\sigma_0 \frac{a}{u_8 \cos\theta_j \sin\theta_j}\,;\quad j = 1,2,3,5,6,7\,. \tag{12}$$

For u_8 = 4 mm, we obtain the solutions neglecting the constraints (4). They are: E_1 = E_7 = 50 GPa; E_2 = E_6 = 54.17 GPa; E_3 = E_5 = 83.33 GPa and E_4 is arbitrarly. For non-defined values of the Young's modulus (σ_4 = 0) we can consider conventionally $E_4 = E_{\min}$.

Using the proposed algorithm, with Eq. (5,a) for α = 1.0, convergence criterion (6,a) with ε_E = 10^{-6}, the results are presented in Fig. 5.



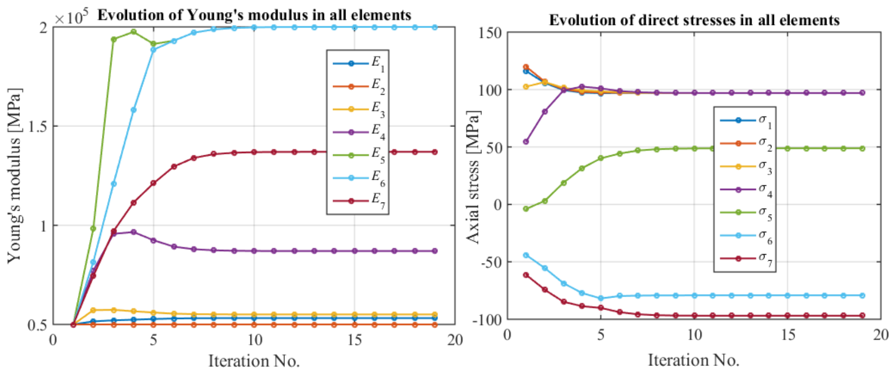
(a) Design variables convergence

(b) Stress convergence

Fig. 5 – Numerical results for Application 1, LC2, considering α = 1.0, Eq. (5,a) and ε_E = 10^{-6}.

Loading case 3 (LC3)

If we consider arbitrarily the forces F_x = F_y = 25 kN as LC3 and using the proposed algorithm, Eq. (5,a) for α = 1.0, convergence criterion (6,a) with ε_E = 10^{-6} and E_0 = 50 GPa, the results are presented in Fig. 6 for the case with $E_{\min}$ = 50 GPa and $E_{\max}$ = 200 GPa. The obtained displacements from numerical analysis were u_8 = 2.5292 mm and v_8 = 1.1143 mm. Without constraints, $E_5 \to \infty$, but the displacements do not change essentially.



(a) Design variables convergence (b) Stress convergence

Fig. 6 – Numerical results for Application 1, LC3, considering α = 1.0, Eq. (5,a), $\varepsilon_E = 10^{-6}$, E_{min} = 50 GPa and E_{max} =200 GPa.

The numerically obtained solutions including the constraints (4), are: E_1 = 53.212 GPa; E_2 = 50.000 GPa; E_3 = 55.027 GPa; E_4 = 86.994 GPa; E_5 = E_6 = 200.00 GPa; E_7 = 137.03 GPa. The stresses result as: $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$ = 96.939 MPa; σ_5 = 48.823 MPa; σ_6 = -79.177 MPa and σ_7 = -96.939 MPa.

5.2. APPLICATION 2: CIRCULAR STRESS RAISER

A circular stress raiser in a thin square plate (state of plane stress) is considered (Fig. 7). The geometry is symmetric and defined by $L_x = L_y$ = 100 mm.

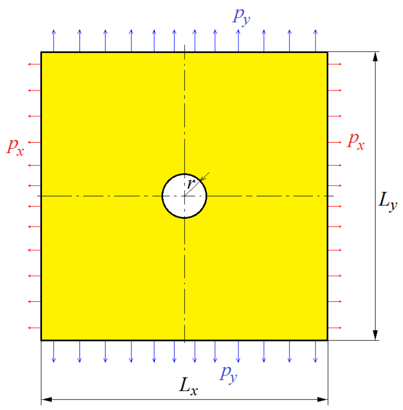


Fig. 7 – A plate with a circular hole.

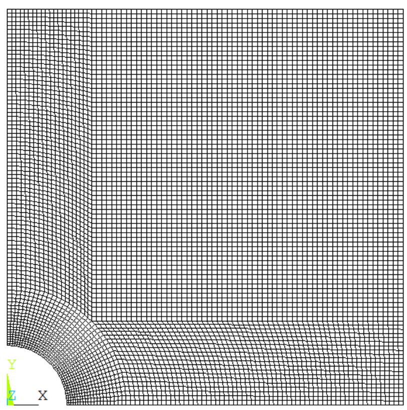


Fig. 8 – Adopted mesh (7534 elements).

The hole is circular with a radius r = 7.5 mm and $p_x = p_y$ = 10 MPa (biaxial uniform loading). Due to symmetry conditions, we use a quarter of the plate for the finite element model, Fig. 8. The used finite elements are quadrilaterals with four nodes (Plane182 in ANSYS), and we adopt a full integration scheme for element stiffness matrix formulation. The initially chosen longitudinal modulus of elasticity is E_0 = 800 MPa and ν = 0.3 = const. We limit the Young's modulus to $E_{min} = E_0$ and $E_{max} = 10 \cdot E_0$. It is required to find the distribution of the modulus of elasticity $E(x,y)$ as to minimize the equivalent von Mises stress (Eq. 2,b).

For α = 0.5 in relation (5,a) convergence is ensured uniformly, that is without oscillations (Fig. 9) and more rapidly than for other values of α. The maximum number of imposed iterations is N_{iter} = 20. The condition of convergence is that from relation (6,b) in which a relatively high value is considered as ε_σ = 0.1.

The final variation of Young's modulus is presented in Fig. 10, and the distribution of the initial von Mises stresses (at iteration 1) and after convergence (at iteration 9) is presented in Fig. 11, respectively Fig. 12. In Fig. 12 the stress distributions are plotted on the deformed configuration geometry, with a displacement scale factor of 50, to better see the contour deformations.

The distribution of $E(x,y)$ shown in Fig. 10 agrees with the results reported in [22], that is the softer material will "gather" where stresses have maximum values, if the material is isotropic. By comparing Fig. 11 with Fig. 12, results a decrease of the maximum stresses by almost two times. From Fig. 10, it is obtained the ratio between the extreme values of Young's moduli as 2137/800 = 2.7, which is a plausible result for usual FGM materials. The results from the last three figures were obtained for constant values on each surface of a finite element.

It must be mentioned that analyses for much more refined meshes were performed and the results were very close to those reported in this paper. If the boundary conditions are applied for periodic structures (the loading faces have only uniform displacements) then the reduction (uniformization) of the stresses can be done to a much lower extent.

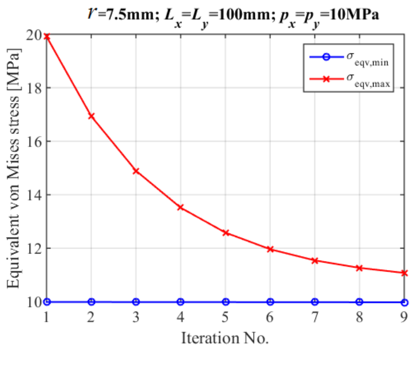


Fig. 9 – Variation of extreme von Mises stresses in Application 2 considering α = 0.5, Eq. (5,a) and $\varepsilon_\sigma = 10^{-1}$.

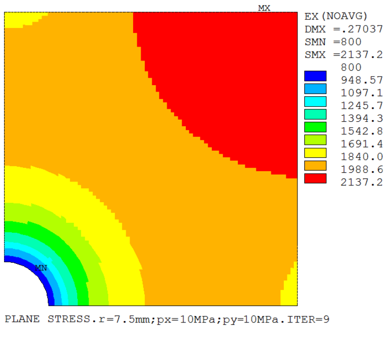


Fig. 10 – Optimized distribution of Young's modulus - $E(x,y)$ [MPa].

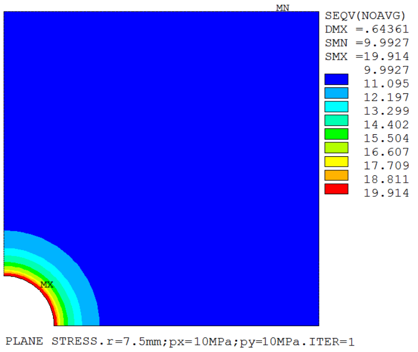


Fig. 11 – Initial distribution of von Mises stresses as $E(x,y) = E_0 = 800$ MPa.

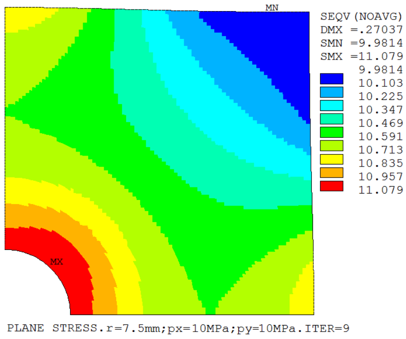


Fig. 12 – Optimized distribution of von Mises stresses σ_{eqv} ($E(x,y)$ in [MPa], corresponding to Fig. 10).

6. CONCLUSIONS

From the applications presented it results that the proposed algorithm is efficient and leads to numerical solutions for problems with or without restrictions which have also an analytical solution. Respectively, for the first application the stress uniformization was done in two variants, with and without imposed restrictions, and the solutions converge towards the analytical solutions. For the second application, for which there is no analytical solution, the stress becomes uniform over the domain of the perforated plate and the Young's modulus is reduced around the hole, as it is known from literature. Besides the applications presented in this paper several other applications were analyzed and the proposed algorithm was validated.

For the situations in which restrictions are encountered in the domain of variation of the modulus of elasticity the results will lead to a considerable reduction of the equivalent stresses. The objective function which was presented in this paper was defined by relation (2,b) following the von Mises theory.

The present researches will continue as to fundament the mathematical aspects and in order to identify new practical applications in which the algorithm can lead to the optimization of the different material distributions.

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