POSITION CONTROL USING GENERALIZED ACTIVE FORCES

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Abstract. Lagrange's equations of motion are employed to find the displacement of the robotics system. The generalized active forces are calculated symbolically in terms of the control torques. To reach the final positions, the PD control torques are improved with a term calculated from the generalized active forces. Robots with one and two arms are analyzed.

Key words: Lagrange's equations, generalized active forces, PID controller.

1. INTRODUCTION

Newton-Euler and Lagrangian equations are most commonly employed to find the motion of kinematic chains. The joint's unknown reaction forces are considered for Newton-Euler equations [1]. The generalized active forces are used with Lagrange, Gibbs-Apell, and Kane methods [2, 3]. The kinematic and dynamic analysis of a robot with linear actuators can be analyzed with the theory of screws and the principle of virtual work [4]. The analysis and control of manipulators concerning their end-effectors were developed in [5]. It is presented a unified approach to motion and force control. A systematic approach for problems with kinematic singularities is analyzed. A recursive Newton-Euler formulation with a computational scheme is developed for industrial robots having three-dimensional closed kinematic chain mechanisms in [6]. Examples are given, and a spatial closed-chain linkage is considered a tree-structured open-chain mechanism with kinematic constraints. Control schemes that generalize the impedance and direct force control are studied in [7]. The control of generalized contact forces is considered for human-robot task compatibility. The ability to control generalized contact motion and force is essential for robot motion. Arimoto presents a critical review of the state of the art of robot control and points out the necessity for improving robot servo-loops [8]. He introduced a quasi-natural potential in Lagrange's formulation of robot dynamics, and the

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hyperstability theoretical framework is applied to the design of control commands. The state-of-the-art discussion of unsolved robot force control problems is presented in [9]. Various impedance strategies and hybrid methods are compared. The issue of stability is discussed. In this study, we start with a proportional derivative controller and compensate for the external forces with a term calculated from the generalized active forces.

2. GENERALIZED ACTIVE FORCES

A system of p particles P_i, i = 1, ..., p in motion is defined by Newton's second law

$$F_i = m_i a_i, \quad i = 1,..., p, \tag{1}$$

where $\mathbf{F}_i$ is net external force acting on P_i, m_i is the mass of P_i, and $\mathbf{a}_i$ is the acceleration of P_i in a fixed reference frame. The inertia force for P_i is defined as principle

$$F_{in\ i} = -m_i a_i, \quad i = 1,.., p \tag{2}$$

and Eq. 1 is written as the expression of D'Alembert's principle

$$F_i + F_{in\ i} = 0, \quad i = 1,..., p\,. \tag{3}$$

The system of p particles is holonomic and has n degrees of freedom. The position vector $\mathbf{r}_i$ of a particle P_i relative to a fixed reference frame is

$$r_i = r_i\left(q_{1,...,}q_n, t\right), \tag{4}$$

where $q_1, ..., q_n$ are the generalized coordinates and t is the time. The generalized coordinates, $q_i(t)$, describe the position of a system, and the generalized velocities, $\dot{q}_i(t)$, are the rate of change of the generalized coordinates with respect to time. The velocity $\mathbf{v}_i$ of P_i is

$$v_i = \sum_{r=1}^{n} \frac{\partial r_i}{\partial q_r}\frac{\partial q_r}{\partial t} + \frac{\partial r_i}{\partial t} = \sum_{r=1}^{n} \frac{\partial r_i}{\partial q_r}\dot{q}_r + \frac{\partial r_i}{\partial t}. \tag{5}$$

Next, replace Eq. (3) with

$$\sum_{i=1}^{p}\left(F_i + F_{in\ i}\right)\cdot\frac{\partial r_i}{\partial q_r} = 0\,. \tag{6}$$

The generalized active force Q_r is defined as

$$Q_r = \sum_{i=1}^{p} \frac{\partial r_i}{\partial q_r} \cdot F_i = \sum_{i=1}^{p} \frac{\partial v_i}{\partial \dot{q}_r} \cdot F_i \,. \tag{7}$$

The generalized inertia force $K_{\mathrm{in}r}$ is defined as

$$K_{\mathrm{in}\,r} = \sum_{i=1}^{p} \frac{\partial r_i}{\partial q_r} \cdot F_{\mathrm{in}\,i} = \sum_{i=1}^{p} \frac{\partial v_i}{\partial \dot{q}_r} \cdot F_{\mathrm{in}\,i} \,, \tag{8}$$

and Eqs. (6) can be written as

$$Q_r + K_{\mathrm{in},\,r} = 0, \;\; r = 1, \ldots, n. \tag{9}$$

Equations (9) are Kane's dynamical equations. Consider the generalized inertia force, $K_{\mathrm{in}r}$

$$\begin{aligned} K_{\mathrm{in}\,r} &= \sum_{i=1}^{p} F_{\mathrm{in}\,i} \cdot \frac{\partial r_i}{\partial q_r} = -\sum_{i=1}^{p} m_i a_i \cdot \frac{\partial r_i}{\partial q_r} = -\sum_{i=1}^{p} m_i \ddot{r}_i \cdot \frac{\partial r_i}{\partial q_r} = \\ &= -\sum_{i=1}^{p} \left[\frac{\mathrm{d}}{\mathrm{d}t} \left(m_i \dot{r}_i \cdot \frac{\partial r_i}{\partial q_r} \right) - m_i \dot{r}_i \cdot \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial r_i}{\partial q_r} \right) \right]. \end{aligned} \tag{10}$$

Now

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial r_i}{\partial q_r} \right) = \sum_{k=1}^{n} \frac{\partial^2 r_i}{\partial q_r \partial q_k} \dot{q}_k + \frac{\partial^2 r_i}{\partial q_r \partial t} = \frac{\partial v_i}{\partial q_r} \,. \tag{11}$$

Furthermore, using Eq. 5:

$$\frac{\partial v_i}{\partial \dot{q}_r} = \frac{\partial r_i}{\partial q_r} \,. \tag{12}$$

Substitution of Eq. 11 and Eq.12 in Eq. 10 leads to

$$\begin{aligned} K_{\mathrm{in}\,r} &= -\sum_{i=1}^{p} \left[\frac{\mathrm{d}}{\mathrm{d}t} \left(m_i v_i \cdot \frac{\partial v_i}{\partial \dot{q}_r} \right) - m_i v_i \cdot \left(\frac{\partial v_i}{\partial q_r} \right) \right] = \\ &= -\left[\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial}{\partial \dot{q}_r} \left(\sum_{i=1}^{p} \frac{1}{2} m_i v_i^2 \right) - \frac{\partial}{\partial q_r} \left(\sum_{i=1}^{p} \frac{1}{2} m_i v_i^2 \right) \right]. \end{aligned} \tag{13}$$

The kinetic energy T of the system in the fixed reference frame is defined as

$$T = \frac{1}{2} \sum_{i=1}^{p} m_i v_i^2 \,. \tag{14}$$

Therefore, the generalized inertia forces are written as:

$$K_{\text{in},\,r} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_r}\right) + \frac{\partial T}{\partial q_r}. \tag{15}$$

The dynamical equations can be written as

$$Q_r - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_r}\right) + \frac{\partial T}{\partial q_r} = 0 \tag{16}$$

,

or

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} = Q_r. \tag{17}$$

The equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} = \sum_{i=1}^{p} \frac{\partial r_i}{\partial q_r} \cdot F_i,\ r = 1,\ldots n, \tag{18}$$

or

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} = \sum_{i=1}^{p} \frac{\partial v_i}{\partial \dot{q}_r} \cdot F_i,\ r = 1,\ldots n, \tag{19}$$

are known as Lagrange's equations of motion of the first kind.

3. ONE ARM ROBOT

A robot with one arm is represented in Fig. 1. The arm is homogeneous with the length L = 1 m and mass m = 5 kg. The gravitational acceleration is g = 9.81 m/s^2. The generalized coordinate q is the angle between link one and the horizontal x-axis. A couple of torque T_{01} is applied to the arm. The initial conditions are: at t = 0 s, $q(0)$ = 0 rad and $\dot{q}(0)$ = 0 rad/s. The final position of the robot is $q_f = \pi/6$ rad. The position vector of the mass center C of the arm is:

$$r_C = 0.5L\cos q(t) i + 0.5L\sin q(t) j \tag{20}$$

and the velocity of C is

$$v_C = \frac{\mathrm{d}r_C}{\mathrm{d}t} = -0.5\cdot L\dot{q}(t)\sin q(t) i + 0.5L\cdot\dot{q}(t)\cos q(t) j \tag{21}$$

.

The angular velocity of the robot is

$$\omega = \dot{q}(t)k\ . \tag{22}$$

The kinetic energy of the robot is

$$T = \frac{1}{2} I_0 \omega \cdot \omega = \frac{m_1 L_1^2}{6} \dot{q}(t)^2\ , \tag{23}$$

where I_0 is the mass moment of inertia about joint *O*.

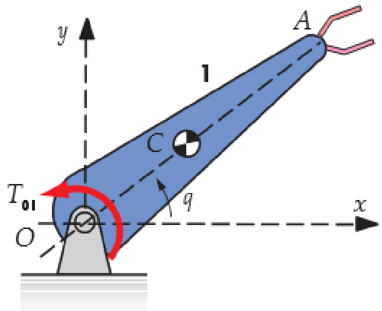


Fig. 1 – One arm robot.

A gravitational force *G* = –*mgj* acts on the robot at *C*. The generalized active force is

$$Q = G \cdot \frac{\partial r_c}{\partial q} + T_{01} \cdot \frac{\partial \omega}{\partial \dot{q}}, \tag{24}$$

Q is symbolically determined in terms of the torque $T_{01}=T_{01}k$ (the moment of the ground 0 on link 1). A control term *R* is defined using the generalized active force

$$R = Q - T_{01}\ . \tag{25}$$

A proportional, derivative (PD) feedback control is selected as

$$T_{01pd} = -K_d \dot{q} - K_p \left(q - q_f\right), \tag{26}$$

where the proportional gain is K_p = 250 Nm/rad and the derivative gain is K_d = 50 Nms/rad. The integrative gain is also arbitrarily selected K_i = 200 Nm/rads. The PID (proportional, integral, derivative) controller is

$$T_{01pid} = -K_d \dot{q} - K_p \left(q - q_f\right) - K_i \int_0^t \left(q - q_f\right) \mathrm{d}u. \tag{27}$$

For a PID controller, the equation of motion are integrodifferential equations and an auxiliary variable is introduced

$$\dot{r}(t) = q(t). \tag{28}$$

The PID controller is

$$T_{01pid} = -K_d \dot{q} - K_p\left(q - q_f\right) - K_i r + K_i \ r \ q_f \ t. \tag{29}$$

Using the generalized active force, Q, the control torque is defined as

$$T_{01Q} = -K_d \dot{q} - K_{p1}\left(q - q_{1f}\right) - R. \tag{30}$$

Figure 2 depicts the angle of the robot using 3 control torques: PD controller, PID controller, and Q controller obtained from generalized active force. Using a PD controller, the final reference position, $q = 30^{\circ}$, is not obtained if external forces are acting on the robot. The final reference position is reached if a PID controller and a Q controller are employed. For the PID controller, there are integrodifferential equations, and for the Q controller, there are only ordinary differential equations.

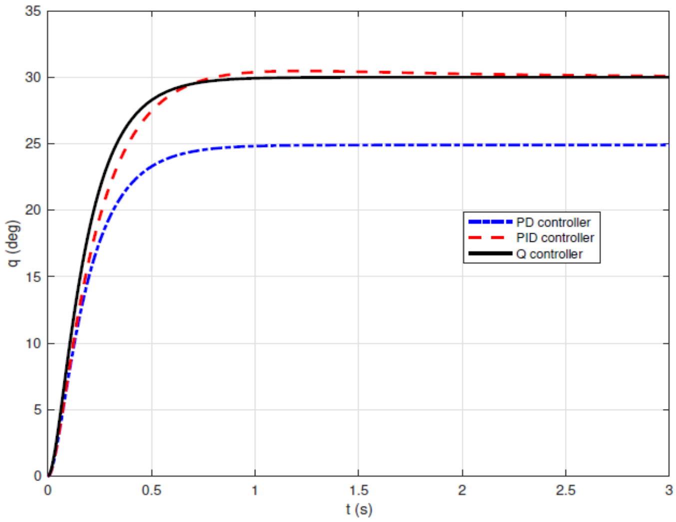


Fig. 2 – PD controller, PID controller and Q controller obtained from generalized active force.

4. TWO-ARM ROBOT

A robot with two elements is shown in Fig. 3. Links 1 and 2 are homogeneous and have the lengths L_1 = 1 m and L_2 = 0.5 m. The masses of the rigid links are m_1 = 5 kg and m_2 = 1 kg. The system's configuration is defined by the relative two generalized coordinates $q_1(t)$ and $q_2(t)$. The generalized coordinate q_1 denotes the radian measure of the angle between link one and the horizontal x-axis and q_2 denotes the radian measure of the relative angle between arm one and arm 2. A couple of torque T_{01} is applied to 1 at O, and a couple of torque $T_{12} = T_{12}\,k$ is applied to 2 by 1 at A. The Cauchy conditions, at $t = 0$ s, are $q_1(0) = q_2(0) = 0$ rad, $\dot{q}_1(0) = \dot{q}_2(0) = 0$ rad/s. The two-arm robot is brought from the initial position to a final position with specified values $q_{1f} = 5\pi/9$ rad and $q_{2f} = -2\pi/3$ rad.

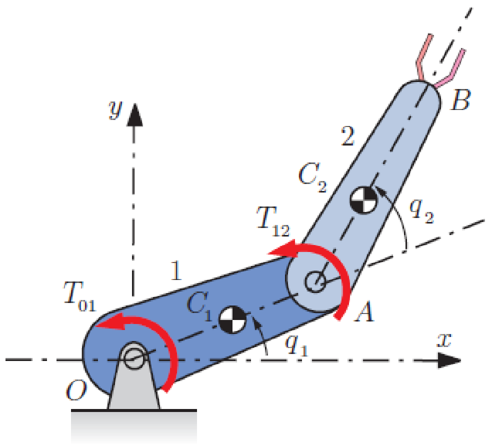


Fig. 3 – Two-arm robot.

The position vector of the mass center of link 2 is r_{C2} and the velocity of C_2 is $v_{C_2} = \dfrac{\mathrm{d}r_{C_2}}{\mathrm{d}t} = \dot{r}_{C_2}$. The angular speed of link 1 is $\omega_1 = \dot{q}_1 k$ and the angular speed of link 2 is $\omega_2 = (\dot{q}_1 + \dot{q}_2)k$.

Kinetic Energy. The kinetic energy of link one, which is in rotational motion, is

$$T_1 = \frac{1}{2} I_O \omega_1 \cdot \omega_1 = \frac{1}{2} I_O \dot{q}_1^2 = \frac{m_1 L_1^2}{6} \dot{q}_1^2 . \tag{31}$$

The kinetic energy of bar 2 is due to the translation and rotation and can be expressed as

$$T_2 = \frac{1}{2} I_{C_2} \omega_2^2 + \frac{1}{2} m_2\, v^2{}_{C_2} = \frac{1}{2} I_{C_2} \left(\dot{q}_1 + \dot{q}_2\right)^2 + \frac{1}{2} m_2 v^2{}_{C_2}, \tag{32}$$

where I_{C_2} is the mass moment of inertia about the center of mass C_2. The total kinetic energy of the system is

$$T = T_1 + T_2 \,. \tag{33}$$

Generalized Active Forces. The gravity forces on links 1 and 2 at the mass centers C_1 and C_2 are $G_1 = -m_1gj$ and $G_2 = -m_2gj$. The generalized force associated with q_1 is

$$Q_1 = G_1 \cdot \frac{\partial r_{C_1}}{\partial q_1} + T_{01} \cdot \frac{\partial \omega_1}{\partial \dot{q}_1} - T_{12} \cdot \frac{\partial \omega_1}{\partial \dot{q}_1} + G_2 \cdot \frac{\partial r_{C_2}}{\partial q_1} + T_{12} \cdot \frac{\partial \omega_2}{\partial \dot{q}_1}. \tag{34}$$

The generalized force associated with q_2 is

$$Q_2 = G_1 \cdot \frac{\partial r_{C_1}}{\partial q_2} + T_{01} \cdot \frac{\partial \omega_1}{\partial \dot{q}_2} - T_{12} \cdot \frac{\partial \omega_1}{\partial \dot{q}_2} + G_2 \cdot \frac{\partial r_{C_2}}{\partial q_2} + T_{12} \cdot \frac{\partial \omega_2}{\partial \dot{q}_2}. \tag{35}$$

The generalized active forces are calculated symbolically in terms of the torques T_{01} and T_{12}. The control terms R_1 and R_2 are calculated from the generalized active forces

$$R_1 = Q_1 - T_{01}, \quad R_2 = Q_2 - T_{12}. \tag{36}$$

Control Torque. The following PD (proportional, derivative) feedback control laws are given

$$\begin{aligned} T_{01\mathrm{pd}} &= -K_{d1} \cdot \dot{q}_1 - K_{p1}\left(q_1 - q_{1f}\right), \\ T_{12\mathrm{pd}} &= -K_{d2} \cdot \dot{q}_2 - K_{p2}\left(q_2 - q_{2f}\right). \end{aligned} \tag{37}$$

The proportional gains are selected as K_{p1} = 794 Nm/rad. and K_{p2} = 57 Nm/rad. The derivative gains are K_{d1} = 126 Nms/rad and K_{d2} = 9 Nms/rad. The integrative gains are arbitrarily assumed K_{i1}=1000 Nm/rads and K_{i2} = 120 Nm/rads.

A linear position controller is a PID (Proportional, integral, derivative) controller with torques

$$\begin{aligned} T_{01pid} &= -K_{d1}\dot{q}_1 - K_{p1}\left(q_1 - q_{1f}\right) - K_{i1}\int_0^t \left(q_1 - q_{1f}\right)\mathrm{d}u. \\ T_{12pid} &= -K_{d2}\dot{q}_2 - K_{p2}\left(q_2 - q_{2f}\right) - K_{i2}\int_0^t \left(q_2 - q_{2f}\right)\mathrm{d}u. \end{aligned} \tag{38}$$

Using a PID controller, the equation of motion are integrodifferential equations. Auxiliary variables are introduced

$$\dot{r}_1(t) = q_1(t), \quad \dot{r}_2(t) = q_2(t) \tag{39}$$

and the controllers are

$$T_{01pid} = -K_{d1}\dot{q}_1 - K_{p1}\left(q_1 - q_{1f}\right) - K_{i1}r_1 + K_{i1}r_1 q_{1f}t,$$
$$T_{12pid} = -K_{d2}\dot{q}_2 - K_{p2}\left(q_2 - q_{2f}\right) - K_{i2}r_2 + K_{i2}r_2 q_{2f}t. \quad (40)$$

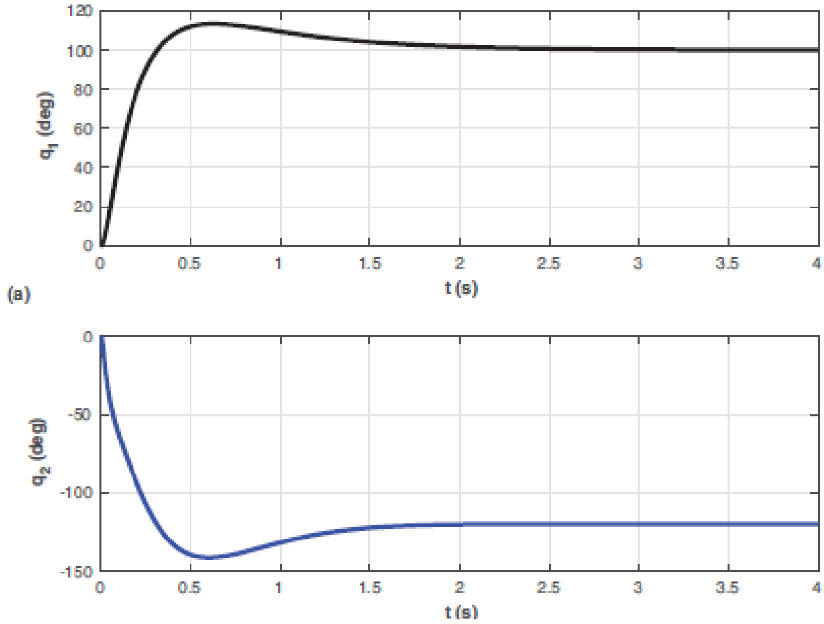
Using the generalized active forces, the control torque are

$$T_{01Q} = -K_{d1}\cdot\dot{q}_1 - K_{p1}\left(q_1 - q_{1f}\right) - R_1,$$
$$T_{12Q} = -K_{d2}\cdot\dot{q}_2 - K_{p2}\left(q_2 - q_{2f}\right) - R_2. \quad (41)$$

Figure 4a shows the solution of the integrodifferential equations using the PID controller and Fig. 4b depicts the angles of the robot using the solution of differential equations obtained from the generalized active forces controller. The angles using both methods reach the final reference positions.

5. CONCLUSIONS

A controller based on generalized active forces is proposed in planar robotic systems with one and two degrees of freedom. We are solving ordinary differential equations and the reference position is quickly reached with this approach. For a PID controller, the integrodifferential equations have to be solved.



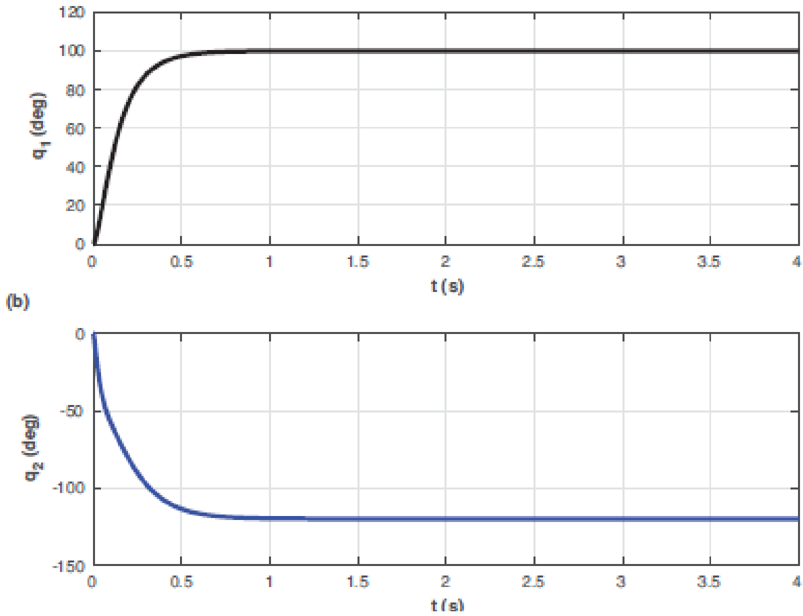


Fig. 4 – a) Solution of integrodifferential equations using PID controller; b) solution of differential equations using torques obtained from generalized active forces.

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