CRACK TIP MECHANICS PROBE OF MULTISCALE CHANGES

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Abstract. Singularity representation at the crack tip and/or smaller region will be considered using six scale transitional physical parameters: three assigned for the nano/micro range ($\mu^*_{na/mi}$, $\sigma^*_{na/mi}$, $d^*_{na/mi}$) and three assigned for the micro/macro range ($\mu^*_{mi/ma}$, $\sigma^*_{mi/ma}$, $d^*_{mi/ma}$). The subscripts nano, micro, and macro are self-evident. Only the ratio of two successive scale sensitive parameters are needed. Although time dependent physical parameters at the lower scale can be found analytical, they regarded as fictitious, mainly because they are not conducible to measurements. The transitional character of multiscale changes according to nano→micro→macro with the respective singularity strength of λ are given by 1.00/0.75/0.50. Since λ=0.5 corresponds to the inverse square root $r^{-0.5}$, where r is the distance from the macro singular point. The micro and nano singular point possess the singularities $r^{-0.75}$ and $r^{-1.00}$, respectively. For example, a critical device component may be designed to operate at the nano/micro/macro scale with a life distribution of $2.5^+/3.5^+/5.5^+$ and total life of 11.5^+ years. Progressive changes are assumed to occur in the direction of nano→micro→macro.

Key words: crack, singular point.

1. INTRODUCTION

Within the context of multi-scaling in time and size [1, 2] and the probing of multi-scale changes, a new paradigm overed not by classical concepts is considered.

1.1. Unit volume energy

The energy in a unit volume of matter $\Delta W/\Delta V$ or $\mathcal{W}$ can be written as

$$\mathcal{W}(t) = \mathcal{A}(t) + \mathcal{D}(t), \;\; D(t) < \mathcal{D}(t)\,, \tag{1}$$

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Ro. J. Techn. Sci. − Appl. Mechanics, Vol. 68, N^{os} 2–3, P. 197–218, Bucharest, 2023

DOI:10.59277/RJTS-AM.2023.2-3.07

while $\mathcal{D}(t)$ is dissipative in character and its rate of change with time remains positive.

1.2. Iso-energy density space

Iso-energy density theory [3] makes use of iso-stress and iso-strain (τ, e) for describing multi-scale changes of multi-dimensions [4, 5]. Measurements of $\mathcal{D}$ have been made by the US Naval Research Laboratory [6].

The nature of the dissipation energy density $\mathcal{D}$ has been discussed and evaluated for many non-equilibrium physical problems [7] and observed to satisfy the condition $\mathrm{d}\mathcal{D}/\mathrm{d}t \geq 0$. That is the slope of the $\mathcal{D}(t)$ versus time t curve remains positive definite. Moreover, the work in [3] has also revealed that the difference of the local behavior of $\mathcal{D}(t)$ in contrast to that of the global behavior. The time rise in the former case is more rapid than that of the latter, especially near a sharp geometric discontinuity. These different variations should be noted.

1.3. Scale transitional behavior

The scale transitional behavior involving macro-micro, micro-nano and nano-pico and so on follows the dual-scale [8–11] transition scheme.

Table 1

Hierarchy of singularities and progressive for multiscale changes
Crack tip mechanics

Scale segmentation	pico/nano/micro	nano/micro/macro	micro/macro/structure
Singularities (order)	1.25/1.00/0.75	1.00/0.75/0.50	0.75/0.50/0.25
Time of arrow (yrs)	$1.5^+ / 2.5^+ / 3.5^+$	$2.5^+ / 3.5^+ / 5.5^+$	$3.5^+ / 5.5^+ / 7.0^+$

Table 1 describes the underlying idea of "crack tip mechanics" that considers the combine use of scale segmentation, singularity strength and time of arrow. Illustrated are the four scales: pico, nano. micro, macro and structure. They are associated with the three groups: pico/nano/micro, nano/micro/macro and micro/macro/structure. Their respective singularities 1.25/1.00/0.75, 1.00/0.75/0.50 and 0.75/0.50/0.25 are representative of the severity of changes and the direction to which they decrease or increase in strength order λ referred to $r^{-\lambda}$ with r being the distance from the singular front. A micro-crack with asymmetry possesses a stronger singular front with $r^{-0.75}$ while the nano-crack with $r^{-1.0}$ pertains to a dislocation type of singularity. A weak singularity $r^{-0.25}$ can be associated with a geometrical defect. The order pico→nano→micro→macro→structure is assumed to be in the direction of decreasing singularity strength. A schematic of singularity strength versus distance r can be found in Fig. 1. The time of arrow in years will depend on the problem definition. An electronic device operating at the pico/nano/micro scale may be designed to have a life distribution of $1.5^{+}/2.5^{+}/3.5^{+}$ for a total life of 7.5^{+} years. Similarly, a component of an instrument may be expected to last for 11.5^{+} years with progressive damage in the order of nano/micro/macro. The same philosophy applies to a critical sub-assembly of a large structure or a component of a high performance device or system. The multiscale internal structure or its components will work in principle by enforcing the pico/nano/micro/macro/structure life time specification followed by manufacturing. However, it suffices to illustrate the principle with a triple scale scheme of nano/micro/macro.

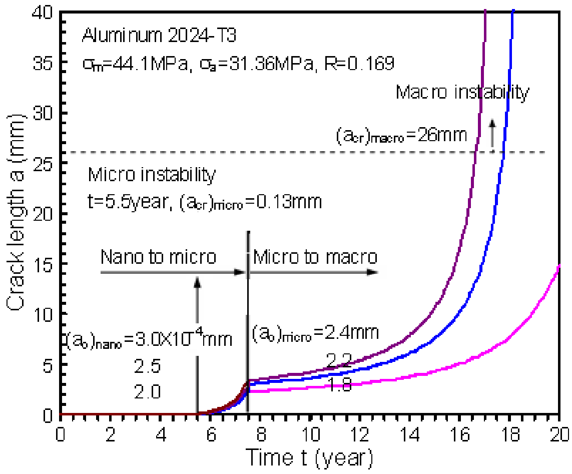

Fig. 1 – Actual time elapsed $1.5/4.0/7.5^{+}$ give intervals for pico/nano/micro as $1.5/2.5^{+}/3.5^{+}$.

Similar representation of diagram from Fig. 1 with actual elapsed time $4.0^{+}/7.5^{+}/13^{+}$ can be used for nano/micro/macro as $2.5^{+}/3.5^{+}/5.5^{+}$, respectively with actual elapsed time $7.5^{+}/13^{+}/20^{+}$ for interval years for micro/macro/structures $3.5^{+}/5.5^{+}/7.0^{+}$.

So (micro/macro/structure) could be obtained for $(0.75/0.50/0.25)$, (nano/micro/macro) for $(1.00/0.75/0.50)$ and (pico/nano/micro) for (1.25/1.00/0.75).

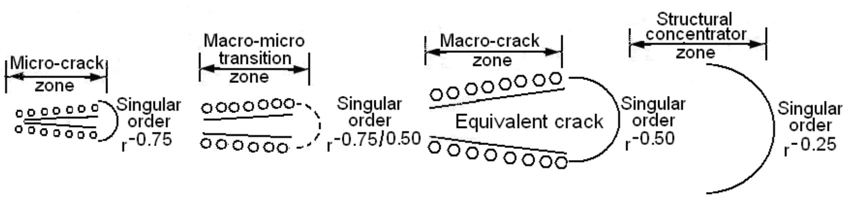

Fig. 2 – Hierarchy of singular behavior (micro/macro/structure) with 0.75/0.50/0.25.

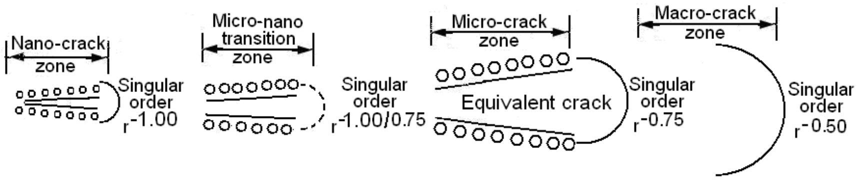

Fig. 3 – Hierarchy of singular behavior (nano/micro/macro) with 1.00/0.75/0.50.

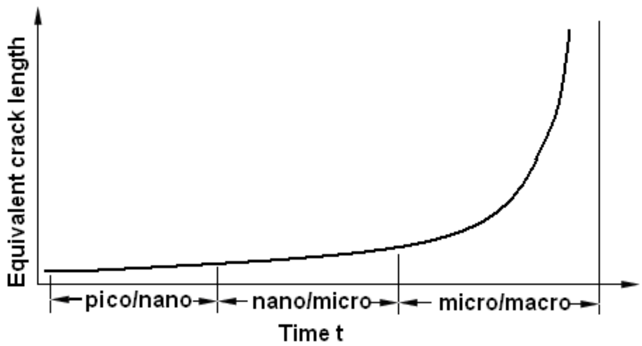

Fig. 4 – Scale transitional behavior of progressive crack growth of arrow Damage by crack growth is assumed to occur progressively in the scale direction of pico→nano→micro→macro coined as the scale direction of arrow.

Figure 4 presents cale transition behavior of progressive crack growth, while Figs. 5 – 12 crack at different scales.

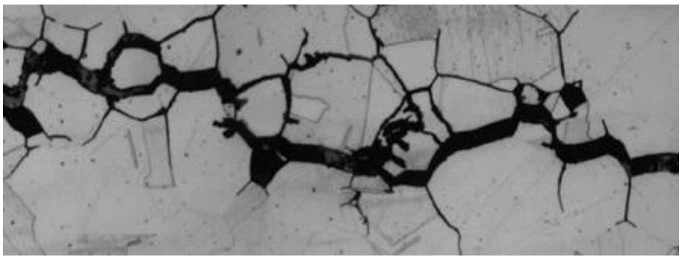

Fig. 5 – In stress corrosion, the cracks follow the interfaces of the crystal grains. The photo shows the intergranular cracks in a real material.

Fig. 6 – Stress corrosion morphology of crack pattern.

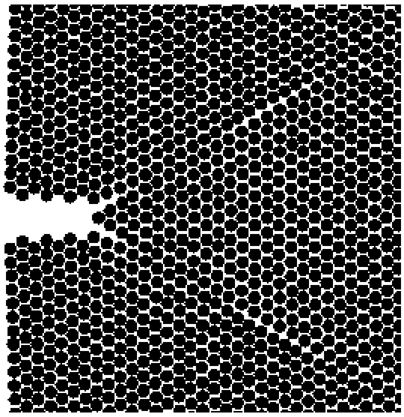

Fig. 7 – Crack branching can be modeled by invoking fracture criterion based on the maximum stress or the minimum strain energy density criterion to determine the direction of crack branching.

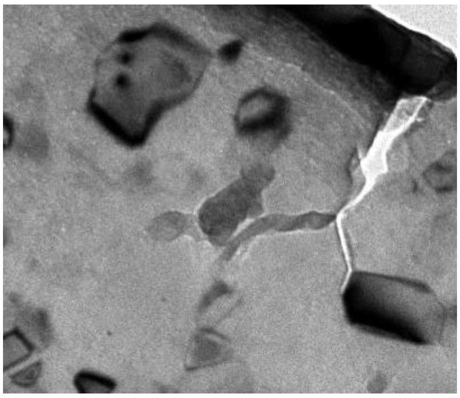

Fig. 8 – Crack would initiate from a void and stop shirt of obstacle and then continue to propagate along the interface.

Fig. 9 – Tate Modern Gallery in London enter the galleries giant turbine hall, confronted are the disturbing sight of a giant crack. Starting as a small fissure, the 167 m fracture grows wider and deeper as it snakes the entire length of the hall.

Fig. 10 – A gap in rock strata could well represent the San Andreas fault in California.

Fig. 11 – An opening magnified could also be mistaken as the San Andreas fault line.

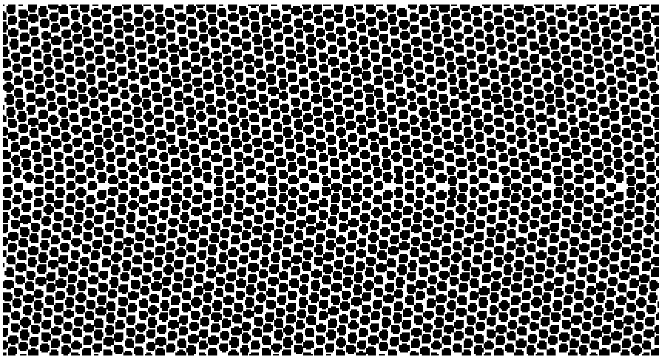

Fig. 12 – Computer simulation of row of atoms.

In the micro/macro and nano/micro dual scale models, the energy density factors can be written as

$$\Delta S_{micro}^{macro} = \Delta r \mathcal{W}_{micro}^{macro} \tag{2}$$

and

$$\Delta S_{micro}^{macro} = \Delta r \mathcal{W}_{nano}^{macro} \tag{3}$$

One may choose $\Delta r = 1\text{mm}$ for Eq. (2) and $\Delta r = 10^{-3}\text{mm}$ or Eq. (3). Note that

$$\Delta S_{micro}^{macro} = a\sigma_a \sigma_m \mu_{mi/ma}^{*} \left(1 - \sigma_{mi/ma}^{*}\right)^2 \sqrt{d_{mi/ma}^{*}} \sqrt{\frac{d_{micro}^{macro}}{r}} \tag{4}$$

and

$$\Delta S_{nano}^{micro} = a\sigma_a \sigma_m \mu_{na/mi}^{*} \left(1 - \sigma_{na/mi}^{*}\right)^2 d_{na/mi}^{*} \frac{d_{micro}^{nano}}{r}, \tag{5}$$

where Δr for micro/macro and nano/micro are selected. ΔS_{micro}^{macro} and ΔS_{nano}^{micro} can be calculated from Eqs. (4) and (5) so that $\mathcal{W}_{micro}^{macro}$ and $\mathcal{W}_{nano}^{micro}$ can be found from Eqs. (2) and (3).

When $\mathcal{W}_{micro}^{macro}$ and $\mathcal{W}_{nano}^{micro}$ for times $t_1, t_2, \ldots$ are known, we can calculate that

$$\mathcal{D}_i = \mathcal{W}_{i+1} - \mathcal{W}_i, \; i = 1,2,3. \tag{6}$$

This leads to know $\mathcal{D}(t)$ as a function of time for the micro/macro and nano/micro cases. Now assume the dissipation density $\mathcal{D}_{micro}^{macro}$ is given by

$$\mathcal{D}_{micro}^{macro} = \eta_{micro}^{macro} \left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2, \tag{7}$$

where $\eta_{micro}^{macro}(t)$ is a micro/macro transitional time dependent dissipation coefficient.

For $\mathcal{D}_{micro}^{macro}$ of the nano/micro transition with the assumption.

$$\mathcal{D}_{nano}^{micro} = \eta_{nano}^{micro} \left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2, \tag{8}$$

where $\eta_{nano}^{micro}(t)$ is a nano/micro transitional time dependent dissipation coefficient. Note that we have already known the crack growth rates for both cases.

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \Psi^{macro}_{micro}\left[\Delta S^{macro}_{micro}\right]^{\psi} \tag{9}$$

and

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \Psi^{micro}_{nano}\left[\Delta S^{micro}_{nano}\right]^{\psi}. \tag{10}$$

Hence, $\eta^{macro}_{micro}(t)$ and $\eta^{micro}_{nano}(t)$ can be found and plotted as a function of time t. Also keep in mind that the energy dissipated may be quantified with more consistency in terms of time/size scale such as nano, micro, macro, etc.

2. MICRO/MACRO CRACK GROWTH

Cross linking dual scale cyclic crack growth models have been discussed in [7] where $a_{nano} \to a_{micro} \to a_{macro}$ were predicted from a knowledge of ΔS^{macro}_{micro} and ΔS^{micro}_{nano}.

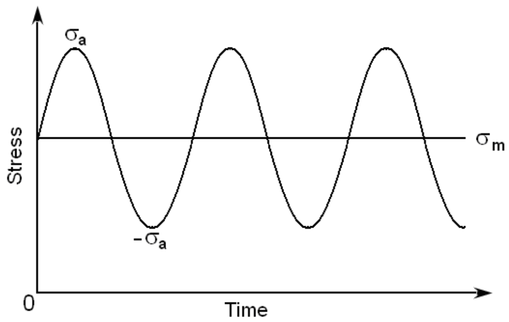

Fig. 13 – Fatigue in fracture mechanics as practiced.

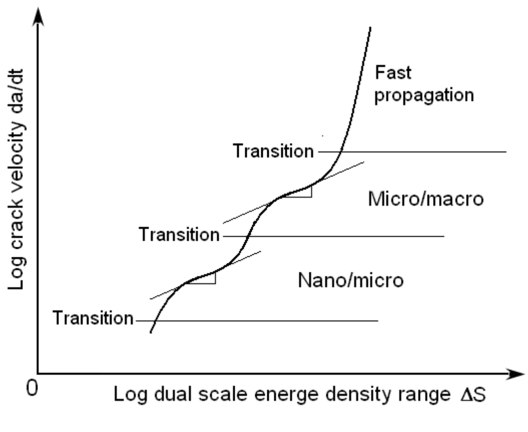

Fig. 14 – Two dual stage nano-, micro- and macro-cracking in fatigue.

2.1. Dual scale micro/macro crack velocity

The two-parameter micro/macro fatigue crack growth model is given by Eq. (9), in which Ψ_{micro}^{macro} accounts for the scale transitional behavior while ψ represents the slope of the da/dt versus ΔS_{micro}^{macro} curve when referred to a log-log plot. The specific form of ΔS_{micro}^{macro} is

$$\Delta S_{micro}^{macro} = a\sigma_a\sigma_m \frac{\mu_{micro}}{\mu_{macro}}\left(1-\frac{\sigma_0^{micro}}{\sigma_\infty^{macro}}\right)^2 \sqrt{\frac{d_{micro}}{r}}\ . \tag{11}$$

Define

$$\mu_{mi/ma}^{*} = \frac{\mu_{micro}}{\mu_{macro}},\ \sigma_{mi/ma}^{*} = \frac{\sigma_0^{micro}}{\sigma_\infty^{macro}},\ d_{mi/ma}^{*} = \frac{d_{micro}}{d_{macro}^{micro}}\ , \tag{12}$$

The fatigue crack growth velocity coefficient Ψ_{micro}^{macro} depends on the macro/micro material property. Also, it is time-dependent. So, Eq. (2) can be written as

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \Psi_{micro}^{macro}(t)\left[a\sigma_a\sigma_m\mu_{mi/ma}^{*}\left(1-\sigma_{mi/ma}^{*}\right)^2\sqrt{d_{mi/ma}^{*}\sqrt{\frac{d_{macro}^{micro}}{r}}}\right]^{\psi} \tag{13}$$

The form of Eq. (13) in the numerical computation is

$$\Delta a_i = \Psi_{micro}^{macro}\left[\sigma_a\sigma_m\mu_{mi/ma}^{*}\left(1-\sigma_{mi/ma}^{*}\right)^2\sqrt{d_{mi/ma}^{*}\sqrt{\frac{d_{macro}^{micro}}{r}}}\right]^{\psi} a_{i-1}^{\psi}\Delta t \tag{14}$$

and

$$a_i = a_{i-1} + \Delta a_i\ . \tag{15}$$

2.2. Micro/macro transitional physical pameters

Three basic parameters $\mu_{mi/ma}^{*}$, $\sigma_{mi/ma}^{*}$ and $d_{mi/ma}^{*}$ are time-dependent. A convex parabola function is assumed.

$$\mu_{mi/ma}^{*}(t) = \frac{\mu_{micro}}{\mu_{macro}} = -0.002t^2 - 0.01t + 3, \quad 0 \le t \le 20 \text{ years}, \tag{16}$$

with $\mu_{mi/ma}^{*} = 3$ for $t = 0$, $\mu_{mi/ma}^{*} = 2.7$ for $t = 10$ and $\mu_{mi/ma}^{*} = 2$ for $t = 20$ years.

$$\sigma^*_{mi/ma}(t) = \frac{\sigma_0^{micro}}{\sigma_\infty^{macro}} = -0.001t^2 + 0.5, \quad 0 \le t \le 20 \text{ years}, \tag{17}$$

with $\sigma^*_{mi/ma} = 0.5$ for $t = 0$, $\sigma^*_{mi/ma} = 0.4$ for $t = 10$ and $\sigma^*_{mi/ma} = 0.1$ for $t = 20$ years.

$$d^*_{mi/ma}(t) = \frac{d_{micro}}{d^{micro}_{macro}} = -0.001t^2 + 5, \quad 0 \le t \le 20 \text{ years}, \tag{18}$$

with $d^*_{mi/ma} = 0$ for $t = 0$, $d^*_{mi/ma} = 4$ for $t = 10$ and $d^*_{mi/ma} = 1$ for $t = 20$ years.

Needed are only the static mechanical properties of 2024-T3 aluminum alloys which are referred to as those at t=0 such as $\mu_0 = 27.56$ (GPa) for the shear modulus or the equivalent of the Young's modulus. It suffices to consider only one material related parameter as in the inter-atomic force model considered.

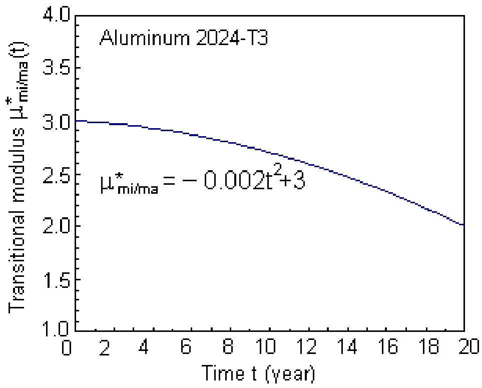

Fig. 15 – Transitional modulus $\mu^*_{mi/ma}(t)$ versus time for 2024-T3.

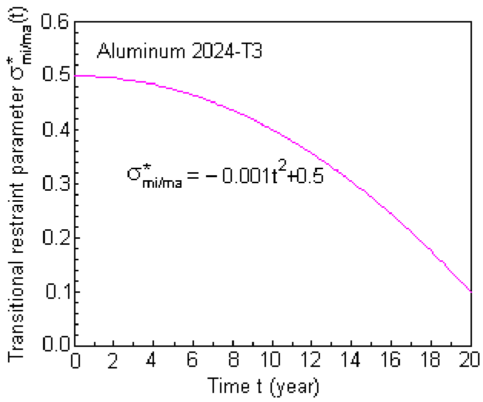

Fig. 16 – Transitional restrain parameter $\sigma^*_{mi/ma}$ versus time t for 2024-T3.

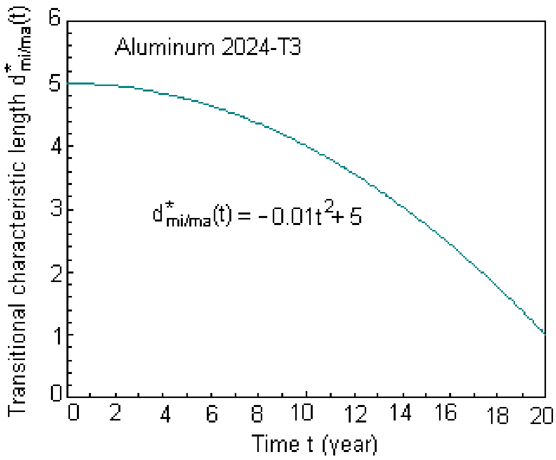

Fig. 17 – Transitional characteristic length $d^*_{mi/ma}(t)$ for 2024-T3.

2.3. Micro/macro transitional physical parameters

A specific form of function $\Psi^{macro}_{micro}(t)$ can be taken as

$$\Psi^{macro}_{micro}(t) = \Psi_{mi/ma}\left(t^3 + At^2 + Bt + C\right), 0 \le t \le 20 \text{ years}, \tag{19}$$

where unit of Ψ^{macro}_{micro} is $\text{mm}^7/\left(N^4\text{year}\right)$. For 2024-T3: $\psi = 2$, $\Psi_{mi/ma} = 2.3602\times10^{-13}$, $A = 2.3810$, $B = 77.7871$, $C = 11966.25$, $d^{micro}_{macro}/r = 1$.

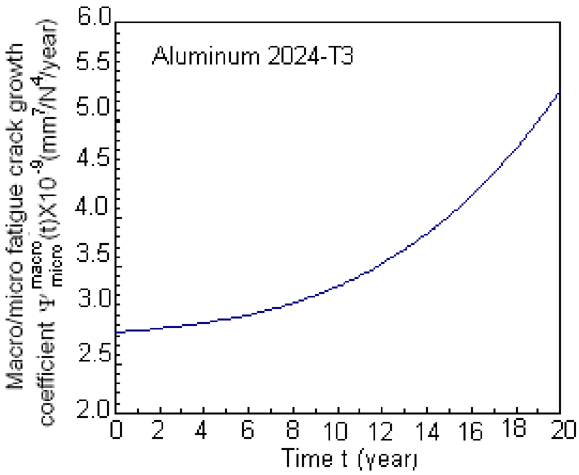

Fig. 18 – Macro/micro cyclic crack velocity Ψ^{macro}_{micro} *vs* time *t* for 2024-T3.

2.4. *Crack growth in the micro/macro scale range*

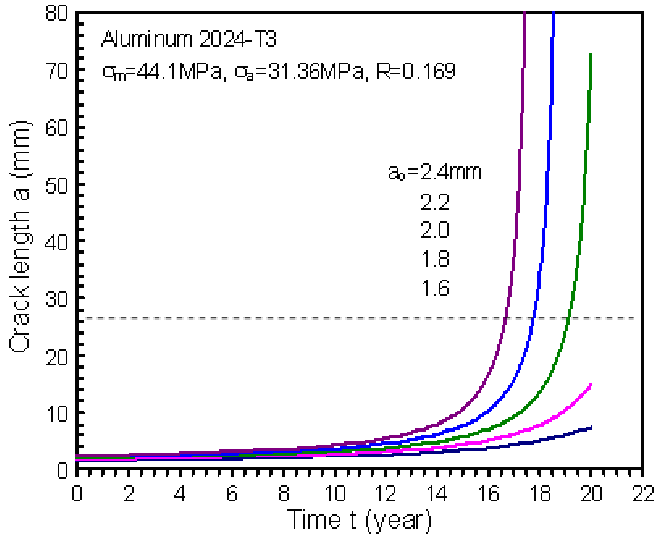

Fig. 19 – Crack length *vs* t for 2024-T3. Critical point at $a_{cr} \approx 26\text{mm}$.

2.5. *Crack growth rate in the micro/macro scale range*

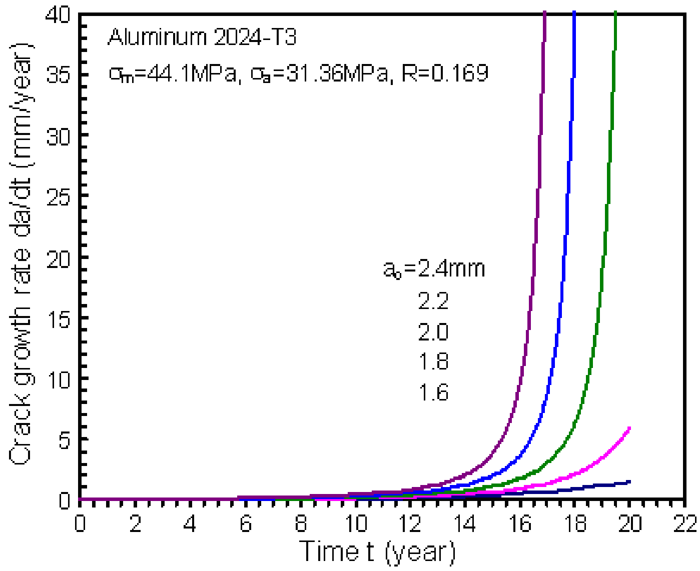

Fig. 20 – Crack growth rate d*a*/d*t* *vs* t for aluminum 2024-T3.

3. NANO/MICRO CRACK GROWTH

3.1. Dual scale nano/micro crack velocity

Suppose that a nano/micro fatigue crack growth model were constructed as

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \Psi_{nano}^{micro}\left[\Delta S_{nano}^{micro}\right]^{\psi}, \tag{20}$$

where

$$\Delta S_{nano}^{micro} = a\sigma_a\sigma_m\mu_{na/mi}^{*}\left(1-\sigma_{na/mi}^{*}\right)^2 d_{na/mi}^{*}\frac{d_{micro}^{nano}}{r} \tag{21}$$

and

$$\mu_{na/mi}^{*} = \frac{\mu_{nano}}{\mu_{micro}},\ \sigma_{na/mi}^{*} = \frac{\sigma_0^{nano}}{\sigma_\infty^{micro}},\ d_{na/mi}^{*} = \frac{d_{nano}}{d_{micro}^{nano}}, \tag{22}$$

such that

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \Psi_{nano}^{micro}\left[a\sigma_a\sigma_m\mu_{na/mi}^{*}\left(1-\sigma_{na/mi}^{*}\right)^2 d_{na/mi}^{*}\frac{d_{micro}^{nano}}{r}\right]^{\psi} \tag{23}$$

or

$$\Delta a_i = \Psi_{nano}^{micro}\left[\sigma_a\sigma_m\mu_{na/mi}^{*}\left(1-\sigma_{na/mi}^{*}\right)^2 \sqrt{d_{na/mi}^{*}}\sqrt{\frac{d_{micro}^{nano}}{r}}\right]^{\psi}\cdot a_{i-1}^{\psi}\Delta t \tag{24}$$

where $a_i = a_{i-1} + \Delta a_i$.

3.2. Nano/micro transitional physical parameters

$$\mu_{na/mi}^{*}(t) = \left(-0.012t^2 - 0.06t + 18\right)\times 10^3,\quad 0 \le t \le 8 \text{ year}, \tag{25}$$

$$\sigma_{na/mi}^{*}(t) = \frac{\sigma_0^{nano}}{\sigma_\infty^{micro}} = -0.001t^2 + 0.5,\quad 0 \le t \le 8 \text{ year}, \tag{26}$$

$$d_{na/mi}^{*}(t) = \frac{d_{nano}}{d_0^{nano}} = \sqrt{-0.001t^2 + 5},\quad 0 \le t \le 8 \text{ year}. \tag{27}$$

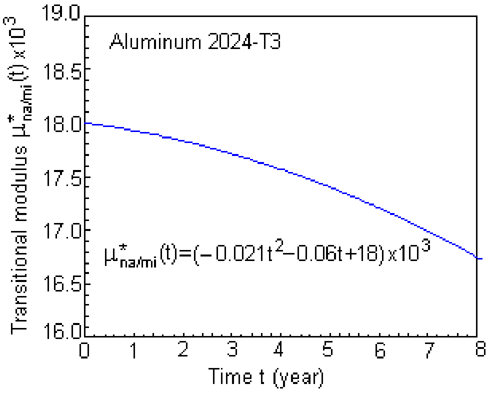

Fig. 21 – Transition modulus $\mu^*_{na/mi}(t)$ *versus* time *t* for 2024-T3.

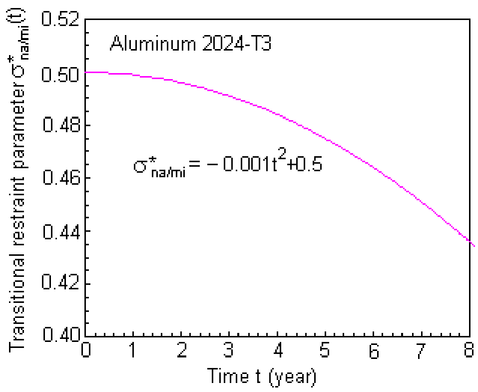

Fig. 22 – Transitional restrain parameter $\sigma^*_{na/mi}(t)$ *versus* time *t* for 2024.

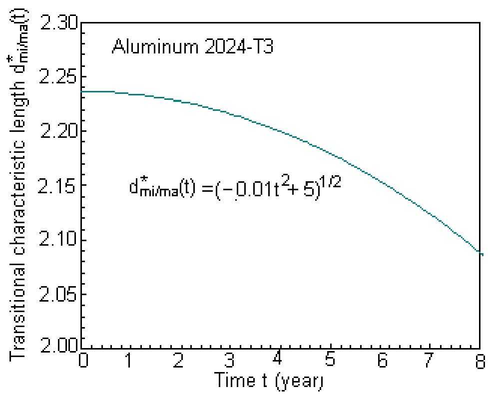

Fig. 23 – Transitional characteristic length $d^*_{na/mi}(t)$ *versus* time *t* for 2024.

3.3. Nano/micro transitional crack velocity

A specific form of function $\Psi_{micro}^{macro}(t)$ can be taken as

$$\Psi_{nano}^{micro}(t) = \Psi_{na/mi}(t+D),\, 0 \le t \le 8 \text{ year.} \tag{28}$$

where unit of Ψ_{nano}^{micro} is $mm^4/\left(N^2 year\right)$ for 2024-T3: $\psi = 1$, $\Psi_{na/mi} = 6.658047\times10^{-9}$, $D = 9.6588$.

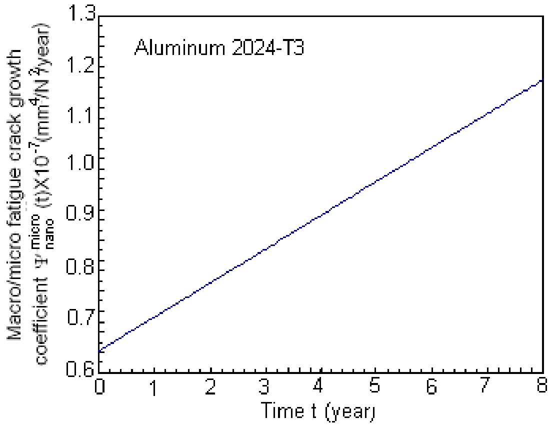

Fig. 24 – Micro/nano cyclic crack speed $\Psi_{nano}^{micro}(t)$ *versus* time t for 2024-T3

3.4. Crack growth in the nano/micro scale range

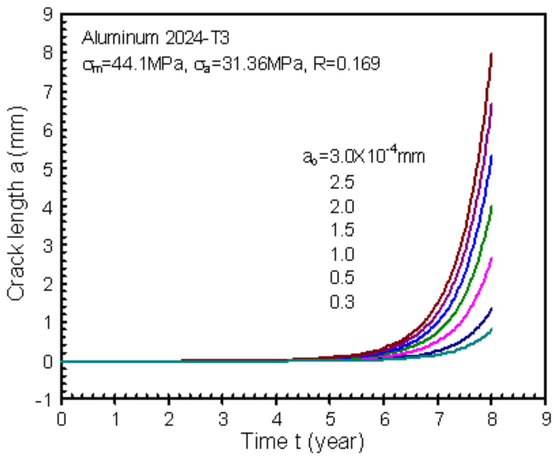

Fig. 25 – Crack growth from nano to micro *versus* time t for 2024-T3.

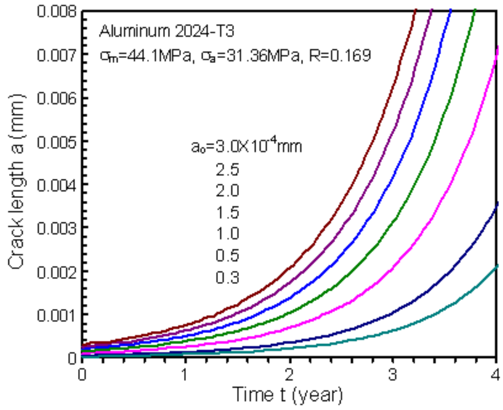

Fig. 26 – Enlarge view of Fig. 16.

3.5. Crack growth rate in the nano/micro scale range

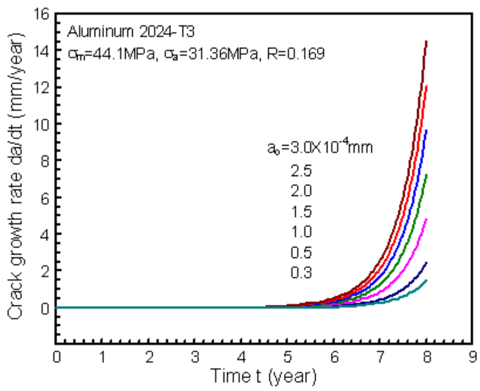

Fig. 27 – Crack growth rate d*a*/d*t* from nano to micro *versus* time *t* for 2024-T3.

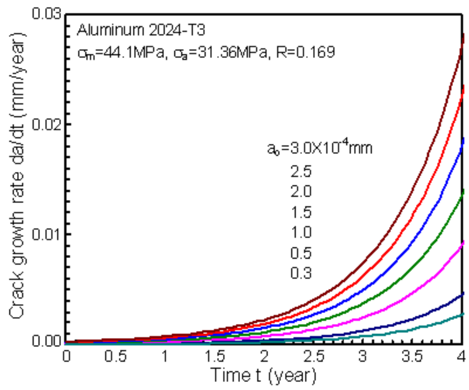

Fig. 28 – Enlarge view of Fig. 18.

4. SCALE TRANSITIONAL BEHAVIOR OF ENERGY DISSIPATION

In the micro/macro and nano/micro dual scale models, the energy density factors can be written as

$$\Delta S_{micro}^{macro} = \Delta r \left(\frac{\Delta W}{\Delta V} \right)_{micro}^{macro} \tag{29}$$

and

$$\Delta S_{nano}^{micro} = \Delta r \left(\frac{\Delta W}{\Delta V} \right)_{nano}^{micro} . \tag{30}$$

One may choose $\Delta r = 1\text{mm}$ for Eq. (2) and $\Delta r = 10^{-3}\text{mm}$ for Eq. (3). Note that using Eq. (4) and (5) where Δr for micro/macro and nano/micro are selected. ΔS_{micro}^{macro} and ΔS_{nano}^{micro} can be calculated from Eqs. (4) and (5) so that $\left(\frac{\Delta W}{\Delta V} \right)_{micro}^{macro}$ and $\left(\frac{\Delta W}{\Delta V} \right)_{nano}^{micro}$ can be found from Eqs. (1) and (2). When $\left(\frac{\Delta W}{\Delta V} \right)_{micro}^{macro}$ and $\left(\frac{\Delta W}{\Delta V} \right)_{nano}^{micro}$ for times $t_1, t_2, \ldots$ are known, we can calculate that:

$$\left(\frac{\Delta D}{\Delta V} \right)_i = \left(\frac{\Delta W}{\Delta V} \right)_{i+1} - \left(\frac{\Delta W}{\Delta V} \right)_i , \; i = 1, 2, 3, \ldots . \tag{31}$$

This leads to know $\left(\frac{\Delta D}{\Delta V} \right)(t)$ as a function of time for the micro/macro and nano/micro cases. Now assume the dissipation density $\left(\frac{\Delta D}{\Delta V} \right)_{micro}^{macro}$ is given by

$$\left(\frac{\Delta D}{\Delta V} \right)_{micro}^{macro} = \eta_{micro}^{macro} \left(\frac{\mathrm{d}a}{\mathrm{d}t} \right)^2 , \tag{32}$$

where $\eta_{micro}^{macro}(t)$ is a micro/macro transitional time dependent dissipation coefficient. The same applies to the nano/micro transition with the assumption:

$$\left(\frac{\Delta D}{\Delta V} \right)_{nano}^{micro} = \eta_{nano}^{micro} \left(\frac{\mathrm{d}a}{\mathrm{d}t} \right)^2 , \tag{33}$$

where $\eta_{nano}^{micro}(t)$ is a nano/micro transitional time dependent dissipation coefficient. Note that we have already know the crack growth rates for both cases given by Eq. (9) and (10).

Hence, $\eta_{micro}^{macro}(t)$ and $\eta_{nano}^{micro}(t)$ can be found and plotted as a function of time *t*.

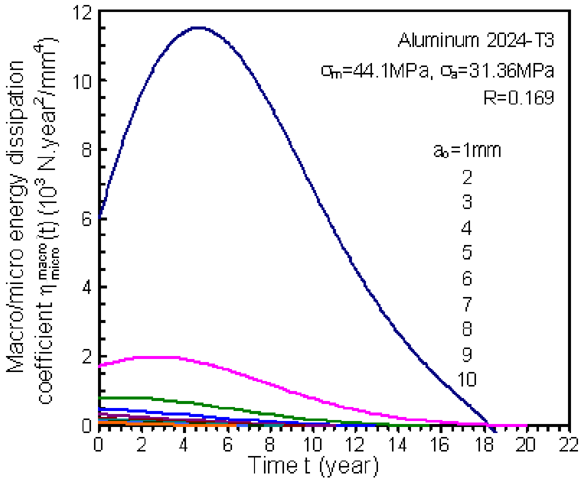

Fig. 29 – Macro/micro energy dissipation coefficient $\eta_{micro}^{macro}(t)$ *versus* time *t* for 2024-T3 with $\sigma_m = 44.1\text{MPa}$, $\sigma_a = 31.36\text{MPa}$, $R = 0.169$.

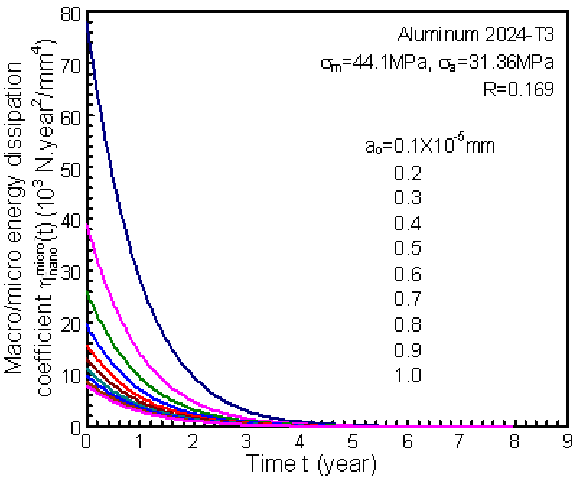

Fig. 30 – Micro/nano energy dissipation coefficient $\eta_{nano}^{micro}(t)$ *versus* time *t* for 2024-T3 with $\sigma_m = 44.1\text{MPa}$, $\sigma_a = 31.36\text{MPa}$, $R = 0.169$.

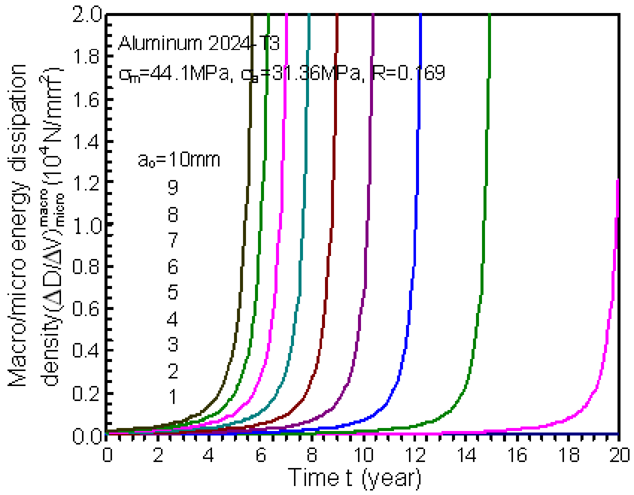

Fig. 31 – Macro/micro dissipation energy density $(\Delta D/\Delta V)_{micro}^{macro}$ *versus* time *t* for 2024-T3 with $\sigma_m = 44.1\text{MPa}$, $\sigma_a = 31.36\text{MPa}$, $R = 0.169$.

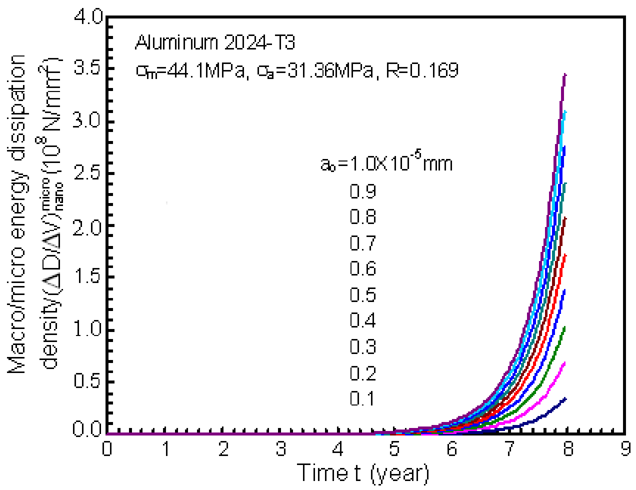

Fig. 32 – Micro/nano dissipation energy density $(\Delta D/\Delta V)_{nano}^{micro}$ *versus* time *t* for 2024-T3 with $\sigma_m = 44.1\text{MPa}$, $\sigma_a = 31.36\text{MPa}$, $R = 0.169$.

5. CONNECTION BETWEEN MACRO/MICRO AND MICRO/NANO

The curves in Fig. 16 are cut at the time $t = 7.5$ year to form Fig. 33.

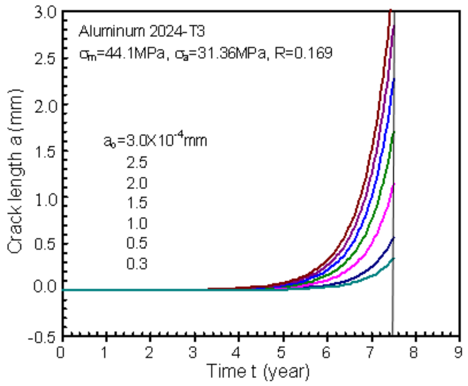

Fig. 33 – Curves in Fig. 18 are cut at t = 7.5 year.

The curves in Fig. 4 are cut at the time t = 7.5 year to form Fig. 34.

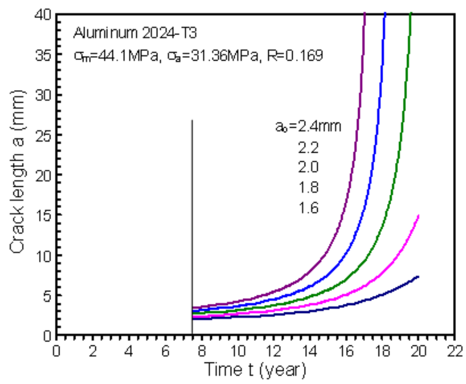

Fig. 34 – Curves in Fig. 6 are cut at t = 7.5 year.

Put Figs. 33 and 34 into one figure to form Fig. 35.

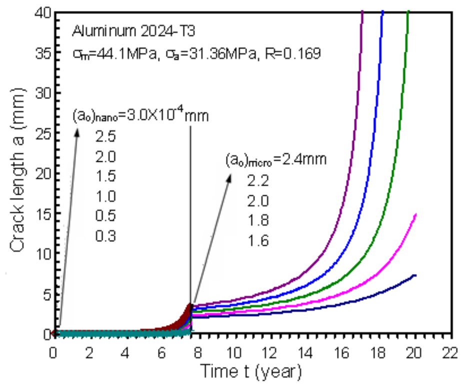

Fig. 35 – Putting Figs. 33 and 34 into one figure.

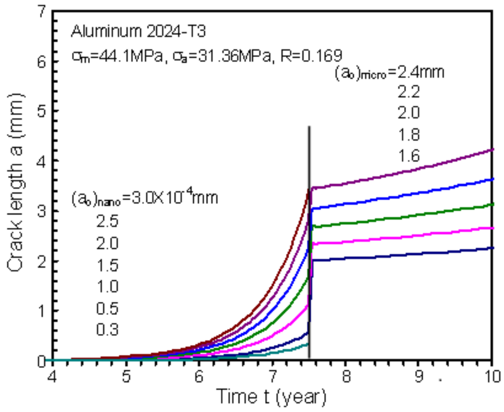

Fig. 36 – An enlarged view of Fig. 35.

Obtain Fig. 37 from Fig. 36 by delete some curves.

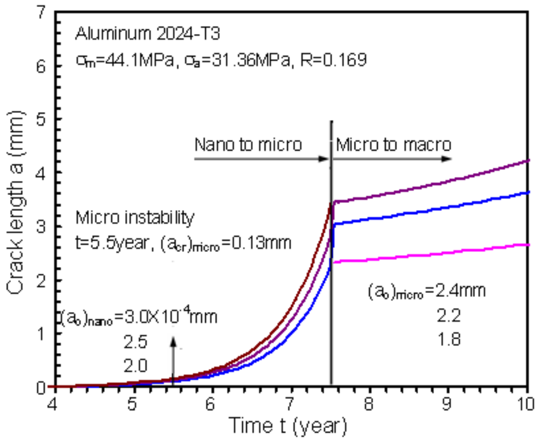

Fig. 37 – Connection between nano to micro and micro to macro.

From Fig. 1 could be obtained the connection between two segments for actual time elapsed $1.5/4.0/7.5^{+}$ and interval years of pico/nano/micro as $1.5/2.5^{+}/3.5^{+}$, respectively for actual time elapsed $4.0^{+}/7.5^{+}/13^{+}$ and interval years of nano/micro/macro as $2.5^{+}/3.5^{+}/5.5^{+}$ and for actual time elapsed $7.5^{+}/13^{+}/18^{+}$ and interval years of micro/macro/structure as $3.5^{+}/5.5^{+}/5.0^{+}$.

Received on August 04, 2023

REFERENCES

1. SIH, G. C., *Implication of scaling hierarchy associated with nonequilibrium: field and particulate*, in: G. C. Sih and V. E. Panin (Eds.), Prospects of meso-mechanics in the 21st century, Special issue of J. of Theoretical and Applied Fracture Mechanics, **37**, pp. 335–369, 2002.
2. SIH, G. C., *Segmented multiscale approach by microscoping and telescoping in material science*, in: G. C. Sih (Ed.), *Multi-scaling in Molecular and Continuum Mechanics: Interaction of Time and Size from Macro to Nano*, Springer, pp. 259–289, 2006.
3. SIH, G. C., *Thermomechanics of solids: nonequilibrium and irreversibility*, J. of Theoretical and Applied Fracture Mechanics, **9**, *3*, pp. 175–198, 1988.
4. SIH, G. C., TZOU, D. Y., *Heating preceded by cooling ahead of crack: macro damage free zone*, J. of Theoretical and Applied Fracture Mechanics, **6**, *2*, pp. 103–111, 1986.
5. SIH, G. C., *Micromehanics associated with thermal/mechanical interaction of polycrystals*, in: G. C. Sih (Ed.), Mesomechanics 2000: *Role of Mechanics for Development of Science and Technology*, Tsinghua University Press, **1**, pp. 143–152, 2000.
6. MAST, P. W., NASH, G. E., MICHOPOULOS, J., THOMAS, R. W., BADALIANCE, R., WOLOCK, I., *Characterization of strain induced damage in composites based on the dissipative energy density – Part I: Basic scheme and formulation; Part II: Composite specimens and naval structures*; *and Part III: General material constitutive relation*, J. of Theoretical and Applied Fracture Mechanics, **22**, *2*, pp. 71–125, 1995.
7. SIH, G C., *Some basic problems in nonequilibrium thermomechanics*, in: Sienietyez S, Salamon P, editors, Flow, Diffusion and Rate Processes; Taylor and Francis, New York, 1992, pp. 218–247.
8. SIH, G. C., TANG, X. S., *Dual scaling damage model associated with weak singularity for macroscopic crack possessing a micro/mesoscopic notch tip*, J. of Theoretical and Applied Fracture Mechanics, **42**, *1*, pp. 1–24, 2004.
9. TANG, X. S., SIH, G. C., *Weak and strong singularities reflecting multiscale damage: micro-boundary conditions for free-free, fixed-fixed and free-fixed constraints*, J. of Theoretical and Applied Fracture Mechanics, **43**, *1*, pp. 1–58, 2005.
10. SIH, G. C., TANG, X. S., *Simultaneous occurrence of double micro/macro stress singularities for multiscale crack model*, J. of Theoretical and Applied Fracture Mechanics, **46**, *2*, pp. 87–104, 2006.