

In memoriam

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BI-MOBILE PLANAR MECHANISMS FOR ROBOTICS OBTAINED THROUGH INVERSE MODELLING

ADRIANA COMANESCU¹, DORU BOBLEA², ANTONIA ONACA³

Abstract. Bi-mobile planar mechanisms are commonly used in various fields of robotics. By adding additional degrees of mobility (rotation and/or translation) to the basis, multi-mobile systems for manipulators or pedipulators (legs) found in various fields are obtained. The paper aims to highlight these mechanisms using bi-mobile planar kinematic linkages described in the classical literature of mechanism theory [1–5,16,17,23,25] in which the basis and the effector are nominated by inverse modelling [8–12,13,15]. The effector extremity of such mechanism may describe any trajectory in its field. This is equivalent to introduce a pair with two constraints which determines the structure to have zero degree of mobility, its inverse structural model could have only passive modular groups. All basis-effector solutions for planar bi-mobile mechanisms obtained from linkages with five degrees of mobility and two and three independent contours are mentioned. The mechanism mobility is given by two active kinematic pairs suitably placed in the same chain [8–15]. The structural algorithm included in the paper offers the possibility to create various optimal solutions for bi-mobile mechanisms. At the same time, the base-effector structural solutions mentioned in the paper are useful in designing these systems, and can be used to verify or adopt appropriate solutions.

Key words: bi-mobile linkage, bi-mobile robot mechanism, optimal solution, baranov truss, passive modular group, Inverse model, direct model, inverse modelling, direct modelling.

1. THEORETICAL CONSIDERATIONS

Any bi-mobile planar mechanism may be obtained from kinematic linkages with five degrees of freedom and consequently with two degrees of mobility. In the traditional literature [1–5, 17, 23, 25, 29] there are mentioned a single linkage with five elements and one independent contour, four linkages with seven elements and two independent loops (Fig. 1) and forty systems L_k with nine links and three independent contours (Table 1).

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The following theorems must be fulfilled to achieve the mechanism model [8, 11]:

- The effector must have a planar motion depending on both independent parameters as to the system has two degrees of mobility;

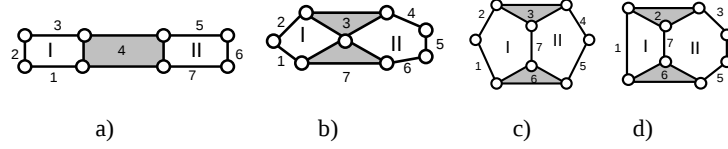
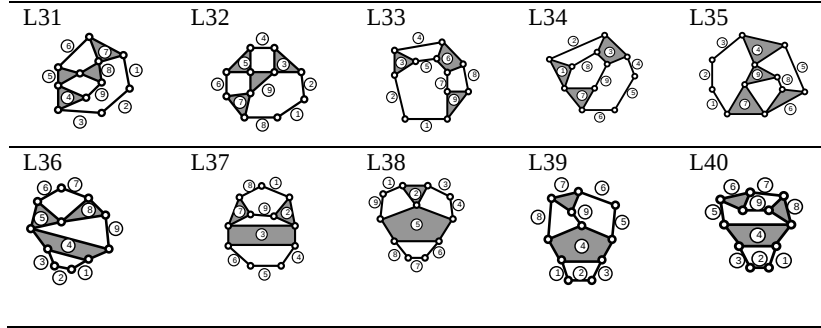


Fig. 1 – Bi-mobile and bi-looped planar kinematic linkages.

Table 1

Bi-mobile and three looped linkages

L1	L2	L3	L4	L5
L6	L7	L8	L9	L10
L11	L12	L13	L14	L15
L16	L17	L18	L19	L20
L21	L22	L23	L24	L25
L26	L27	L28	L29	L30



- The effector cannot belong to a four-bar linkage, Watt or Stephenson structures (Fig. 2);

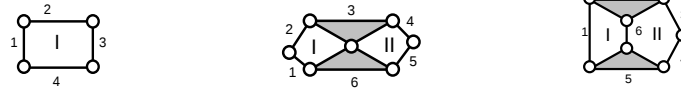


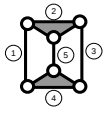
Fig. 2 – Four-bars linkage, Watt and Stephenson linkage.

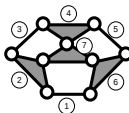
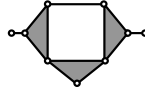
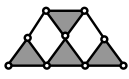
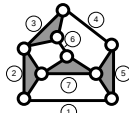
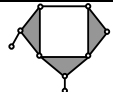
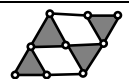
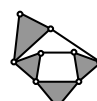
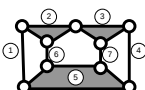
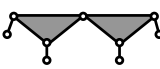
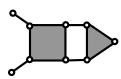
The possible solutions for basis-effector in a linkage L_k with n elements may be mentioned in $A(i, j)$ square and symmetrical matrix ($n \times n$) with Boolean values components;

- Each solution for basis-effector must be verified through its inverse model with zero degrees of mobility. Imposing any position on a trajectory for the effector extremity is equivalent to introduce a kinematic pair with two constrains between the link extremity and the basis, which determines zero mobility for the structure.
- Consequently, the inverse model must have only passive modular groups (GMP) named Assur groups [2, 5, 17, 23] derived from Baranov trusses with 3, 5, 7 elements (Table 2).

Table 2

Passive modular groups (Assur groups)

Baranov truss (BT)	Passive modular group (GMP)
 BT 1	 GMP 1
 BT 2	 GMP 2
	 GMP 3

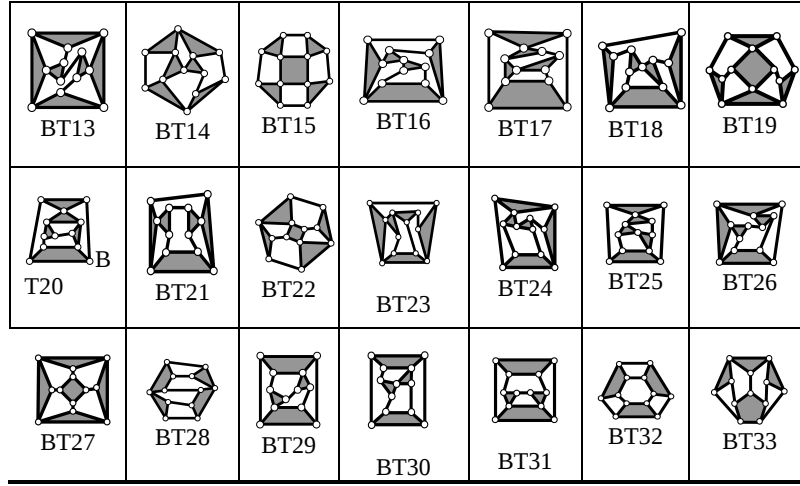
 BT 3	 GMP 5
	 GMP 9
	 GMP 11
 BT 4	 GMP 6
	 GMP 8
	 GMP 10
	 GMP 12
 BT 5	 GMP 4
	 GMP 7
	 GMP 13

The best solutions for basis-effector include a minimum number of passive groups, ideally a single one. In this last case the inverse model is a Baranov truss (Table 3).

Table 3

Baranov trusses for robot mechanisms modeling

 BT6	 BT 7	 BT8	 BT9	 BT10	 BT11	 BT12
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The robot mechanism is finally obtained by introducing two active pairs in the linkages structure through its direct model [12, 13, 15].

The structural algorithm previously mentioned is synthetized in Fig. 3. Here the authors only developed the first part of the algorithm concerning the inverse model optimization.

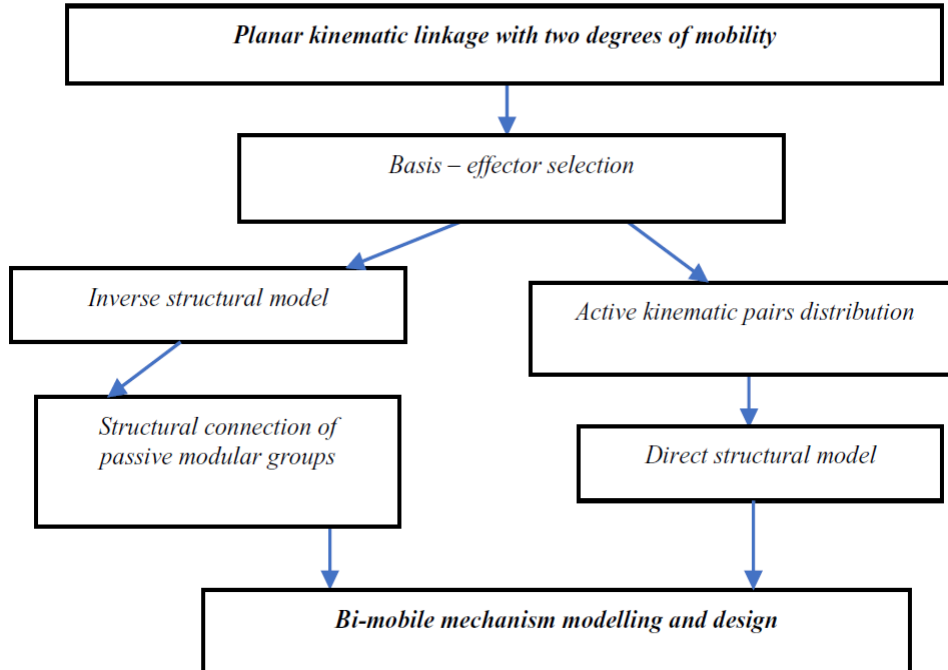


Fig. 3 – Structural algorithm for bi-mobile robot mechanism modelling.

A final remark concerns the component of inverse models associated with chains made according to the theorems outlined above. They consist of passive modular groups with a structure as follows:

- For chains with 7 elements, inverse models could have the following passive modular structures: with 2+2+2, 2+4, 4+2, 6 elements (Table 2);
- For chains with 9 elements, modular passive structures can have respectively 2+2+2+2 (Appendix 1), 2+2+4 (Appendix 2), 2+4+2 (Appendix 3), 4+2+2 (Appendix 4), 2+6 (Appendix 5), 6+2 (Appendix 6), 4+4 (Appendix 7), 8 elements.

2. OPTIMAL INVERSE MODELS FOR BI-MOBILE AND BI-LOOPED PLANAR LINKAGE

A classic example of a mechanism with two degrees of mobility and two independent contours is the Stewart's foot (Fig. 4a), which often equips stepping platforms. The mechanism has 6 moving elements and 8 kinematic pairs of rotation and translation, so that the degree of mobility is $M = 2$. The kinematic translation pairs (1, 2) and (4, 5) are active pairs that ensure well-defined system movement. The fundamental decomposable plane kinematic chain (Fig. 4b), which formed the basis of the direct structural model, is shown in Fig. 5a.

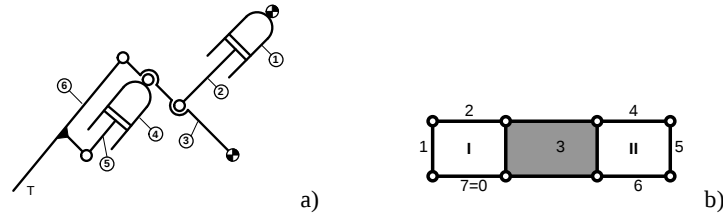


Fig. 4 – Stewart foot.

Point T belonging to execution element 6 not adjacent to basis 7 and not belonging to the same quadrilateral contour as the basis (Fig. 4a), has a trajectory dependent on the parameters of both active translation pairs. Consequently, *the direct structural model* of the mechanism is given in Fig. 4a, to which corresponds the connection of the modular groups in Fig. 5b. It consists of two active modular groups of RTRR type linked in parallel to the basis $0 = 7$ of the mechanism.

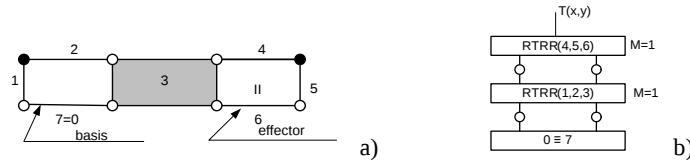


Fig. 5 – Direct structural model of Stewart foot.

Consequently, the *direct structural model* highlights the two parameters of the tracer point T according to the two parameters of the active pairs. Imposing successive point positions on the T point in relation to the fixed element is equivalent to introducing an instantaneous kinematic pair with two restrictions $k = 2$ between the execution element and the basis. In this case, the degree of mobility instantly becomes $M_{inst} = 3m - 2(c + 1)$ i.e. $M_{inst} = 0$ and is characteristic of the *inverse structural model of the mechanism*, Fig. 6 a), which corresponds to the connection of the modular groups in Fig. 6 b).

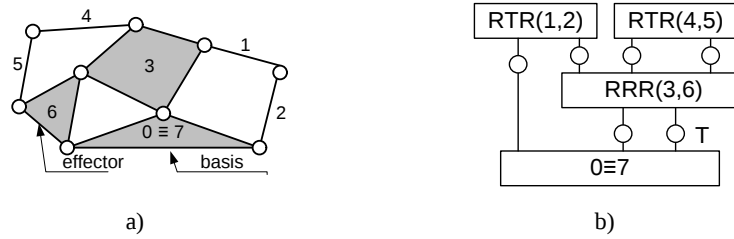


Fig. 6 – Inverse structural model of Stewart foot.

The bi-mobile and bi-looped planar linkages (Fig. 1), may be characterized by their classes of the independent contours (loops). The linkages from Fig. 1a, b, c, d are described as follows: 4–4, 4–5, 5–5, and 4–6.

Using the previously mentioned theorems (§1) the basis-effector matrix with Boolean values for each linkage is important to be correctly achieved. Its matrix element $A(i, j)$ gives the basis i and the effector j . The matrix $(n \times n)$, n being the number of the linkage links is square and symmetrical one with zero values for the principal diagonal.

In the case of the 4–4 linkage (Fig. 1a), the matrix $A(i, j)$ is as follows:

element	1	2	3	4	5	6	7
1	0	0	0	0	1	1	1
2	0	0	0	0	1	1	1
3	0	0	0	0	1	1	1
4	0	0	0	0	0	0	0
5	1	1	1	0	0	0	0
6	1	1	1	0	0	0	0
7	1	1	1	0	0	0	0

For the first linkage 4-4 (Fig. 7), the inverse models [23, 24] reveal only one optimal solution (Fig. 7a), for $A(2,6)$, which is equivalent to Baranov truss BT5 (Fig. 7b). By eliminating basis 2 in the inverse model the classical modular passive group GMP7 is found (Fig. 7c).

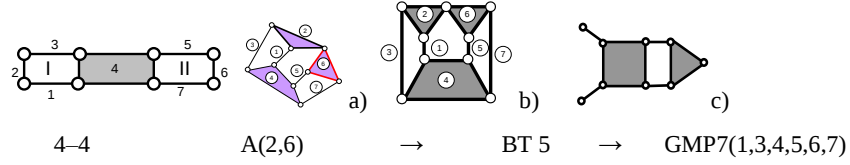


Fig. 7 – Structural modelling for the bi-mobile and bi-looped linkage 4-4.

Likewise, for the bi-mobile linkages 4-5, 5-5, 4-6 (Fig. 1b, c, d) the solutions for optimal structures are found in Table 4.

Table 4

Optimal solutions for linkages 4-5, 5-5 and 4-6

4-5	A(1,5)	BT4	GMP8(2,3,4,5,6,7)
	A(5,1)	BT4	GMP6(1,2,3,4,6,7)
5-5	A(1,4)	BT4	GMP8(2,3,4,5,6,7)
	A(1,4)	BT3	GMP9(2,3,4,5,6,7)
4-6	A(4,1)	BT3	GMP6(1,2,3,4,6,7)

Their inverse models are characterized by passive modular groups GMP with 6 links known from mechanisms literature [2, 5, 23] and derived from previously mentioned Baranov trusses, Table 3. These mechanisms with basis-effector solutions mentioned in Table 4 are also optimal from technological and precision point of view.

3. OPTIMAL INVERSE MODELS FOR BI-MOBILE AND THREE-LOOPED PLANAR LINKAGES

The structural algorithm presented above, Fig. 3 for determining by inverse modelling the basis-effector solutions for bi-mobile and three-contour mechanisms highlights a number of 941 solutions, the models having 9 elements consisting of passive modular groups with the following characteristics of the elements (Table 2): 2+2+2+2, 2+4+2, 2+2+4, 4+2+2, 4+4, 2+6, 6+2, 8 details of which are shown in the paper appendixes.

The optimal basis-effector solutions for the non-decomposable linkages (L1-L35) shown in Table 1 are mentioned in Table 5.

Table 5

Basis-effector solutions for non-decomposable kinematic linkages

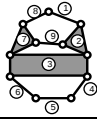
N o	BT	The bi-mobile and three looped planar linkages Optimal solution for the basis-effector									
1.	BT 6	L3				L9					
		A(3,7)		A(7,3)		A(2,6)					
2.	BT 7	L13		L15		L27		L35			
		A(4,8)		A(4,8)	A(8,4)	A(1,7)		A(2,8)	A(8,2)		
3.	BT 8	L2		L5		L15		L28			
		A(2,8)		A(3,7)	A(7,3)	A(2,5)	A(5,2)	A(3,7)	A(7,3)		
4.	BT 9	L6		L29		L30		L31			
		A(1,6)	A(6,1)	A(1,7)	A(1,4)	A(1,6)	A(6,1)	A(2,6)	A(6,2)		
5.	BT 10	L2		L3		L20		L30			
		A(1,6)	A(6,1)	A(2,6)	A(6,2)	A(1,5)	A(5,1)	A(3,6)	A(6,3)		
		L35									
		A(2,5)				A(5,2)					
6.	BT 11	L13									
		A(3,7)									
7.	BT 12	L13		L14		L24		L25			
		A(3,8)		A(4,9)	A(9,4)	A(6,7)	A(7,6)	A(2,7)	A(7,2)		
8.	BT 13	L20		L25		L32					
		A(2,5)		A(1,4)		A(4,1)		A(1,4)		A(4,1)	
9.	BT 14	L3		L5		L11					
		A(3,6)		A(6,3)		A(3,8)		A(8,3)		A(1,6)	A(6,1)
		L14		L19		L30					
		A(1,4)		A(4,1)		A(1,7)		A(7,1)		A(3,6)	

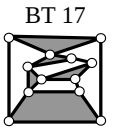
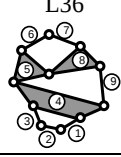
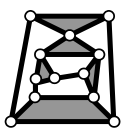
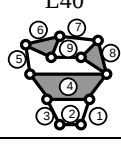
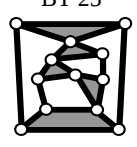
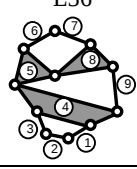
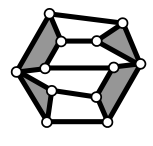
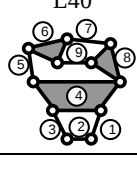
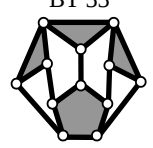
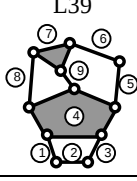
10.	BT 15	L32							
		A(1,9)				A(9,1)			
11.	BT 16	L17		L19			L28		
		A(2,6)	A(6,2)	A(1,8)		A(8,1)	A(2,5)		
12.	BT 17	L18		L29		L33		L34	
		A(3,9)	A(3,9)	A(1,5	A(5,1)	A(1,4)	A(4,1)	A(5,9)	A(9,5)
13.	BT 18	L7		L30		L31		L35	
		A(1,7)	A(7,1)	A(4,8	A(8,4)	A(2,5)	A(5,2)	A(2,9)	A(9,2)
14.	BT 19	L26		L27			L32		
		A(1,6)	A(6,1)	A(3,6)		A(6,3)	A(1,5)		A(5,1)
15.	BT 20	L20		L23			L26		
		A(2,6)	A(6,2)	A(2,6)		A(6,2)	A(3,6)		A(6,3)
16.	BT 21	L21				L25			
		A(1,6)		A(6,1)		A(2,5)		A(5,2)	
17.	BT 22	L8				L30			
		A(1,6)		A(6,1)		A(4,7)		A(7,4)	
18.	BT 23	L7		L10		L14		L35	
		A(2,7)	A(7,2)	A(1,6)	A(6,1)	A(2,5)	A(5,2)	A(2,6)	A(6,2)
19.	BT 24	L8		L10		L12		L15	
		A(2,6)	A(6,2)	A(2,6)	A(6,2)	A(1,4)	A(4,1)	A(1,4)	A(4,1)
20.	BT 25	L17				L20			
		A(1,6)		A(6,1)		A(1,6)		A(6,1)	
21.	BT 26	L21		L23		L27		L34	
		A(2,6)	A(6,2)	A(1,4)	A(4,1)	A(1,4)	A(4,1)	A(2,5)	A(5,2)
22.	BT 27	L12							
		A(1,5)							
23.	BT 29	L26							
		A(2,6)				A(6,2)			
24.	BT 30	L22				L23			
		A(1,5)		A(5,1)		A(1,5)		A(5,1)	
25.	BT 31	L34							
		A(1,5)				A(5,1)			

Applying the same structural procedure the basis-effector optimal solutions for decomposable linkages are shown in Table 6.

Table 6

Basis-effector solutions for decomposable kinematic linkages

No.	BT	The bi-mobile and three looped planar linkages Optimal solution for the basis-effector	
1.	BT 16	L37	
			
		A(1,5)	A(5,1)

2.		
	A(2,6)	A(6,2)
3.		
	A(2,7)	A(7,2)
4.		
	A(2,7)	A(7,2)
5.		
	A(2,6)	A(6,2)
6.		
	A(2,6)	A(6,2)

The optimal solutions for basis-effector selected through Baranov trusses [(BT6-BT33) – (BT32)] have in view only 27 of them, the 28th is a particular one (BT 32) with symmetry properties.

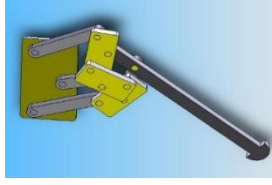


Fig. 8 – Manipulator with the inverse model defined by Baranov truss BT 24 from the linkage L15 with the basis 1 and the effector 4 (Table 5).

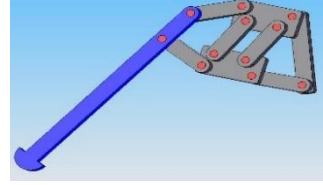


Fig. 9 – Manipulator with the inverse model defined by Baranov truss BT 30 from the linkage L22 with the basis 1 and the effector 5 (Table 5).

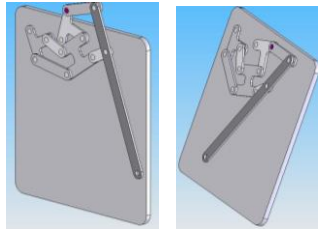


Fig. 10 – Manipulator with the inverse model defined by Baranov truss BT 20 from the linkage L26 with the basis 3 and the effector 6 (Table 5).

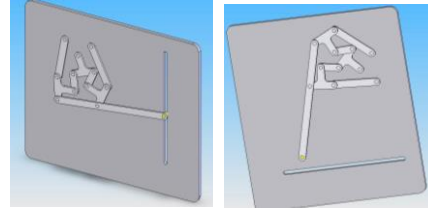


Fig. 11 – Manipulator with the inverse model defined by Baranov truss BT 19 from the linkage L27 with the basis 6 and the effector 3 (Table 5).

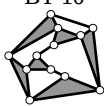
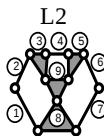
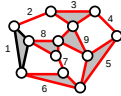
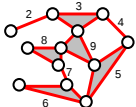
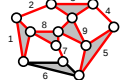
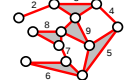
The experimental robot mechanisms (Figs. 8–11) are achieved by means of previously presented theoretical researches.

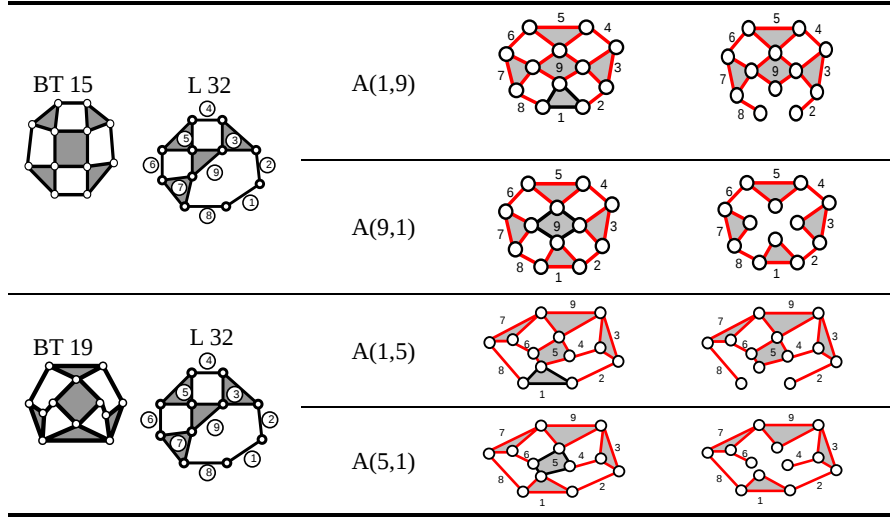
4. PASSIVE MODULAR GROUPS IDENTIFIED BY MEANS OF INVERSE MODELS FOR BI-MOBILE AND THREE-LOOPED PLANAR LINKAGES

Most of the solutions for inverse models from Table 4 and Table 5 give new distinct passive modular groups with eight elements. Some significant examples are mentioned in Table 7, others will be presented in a future paper.

Table 7

Passive modular groups with 8 elements

Baranov truss	Linkage	Basis-effector solution	Inverse model	Passive modular group (new configuration)
		A(1,6)		
		A(6,1)		



5. CONCLUSIONS

The paper makes available to researchers and designers of robotic mechanical systems the assembly of bi-mobile and bi-contour mechanisms obtained from kinematic chains with five degrees of freedom. By nominating the base and effector, the correct and chosen inverse modular structure is ensured so as to achieve optimal control of the structure. A small number of passive modular groups in the reverse model influence the accuracy of motion and its control through a small number of sensors.

This consideration justifies the development of recent research on Baranov trusses with 11 and 13 elements [6, 18, 19, 27, 28, 30]. Further research will focus on minimizing the number of active modular groups in these mechanisms, which raise the performance of precision and control of movements.

The paper synthesizes the all bi-mobile mechanisms with two and three independent contours for robotic systems, being useful in design, similar to the famous atlas J.A.Hrones and G.L.Nelson, dedicated to the synthesis of four-bar mechanisms according to their coupler curves [21].

Received on March 12, 2024

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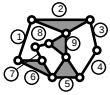
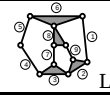
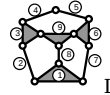
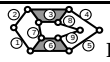
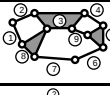
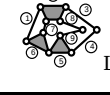
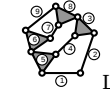
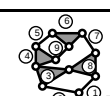
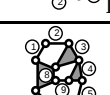
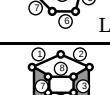
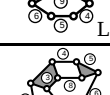
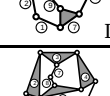
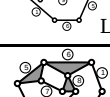
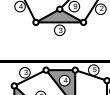
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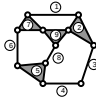
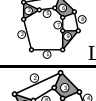
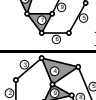
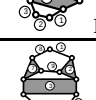
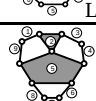
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Appendix 1

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	A(3,7)		A(7,3)		A(6,8)				A(8,6)		A(6,9)			
	A(1,3)		A(3,1)		A(1,9)		A(9,1)		A(2,4)		A(4,2)			
	A(2,8)		A(8,2)		A(2,9)		A(9,2)		A(3,8)		A(8,3)			
	A(1,5)		A(5,1)		A(1,6)		A(6,1)		A(1,9)		A(9,1)			
	A(9,3)		A(4,6)		A(6,4)		A(5,7)		A(7,5)		A(6,8)			
	A(1,6)		A(6,1)		A(2,4)		A(4,2)		A(3,5)		A(5,3)			
	A(6,3)		A(4,6)		A(6,4)		A(4,7)		A(7,4)		A(5,7)			
	A(1,3)		A(3,1)		A(1,5)		A(5,1)		A(1,6)					
	A(1,7)		A(7,1)		A(2,9)		A(9,2)		A(7,9)		A(9,7)			
	A(1,8)		A(8,1)		A(2,4)		A(4,2)		A(3,5)		A(3,8)			
	A(8,3)		A(4,8)		A(8,4)									
	A(1,8)		A(8,1)		A(3,8)		A(8,3)		A(4,6)		A(6,4)			
	A(5,8)		A(8,5)		A(6,8)		A(8,6)		A(6,9)		A(9,6)			
	A(3,5)		A(5,3)		A(4,6)		A(6,4)		A(4,7)		A(7,4)			
	A(7,5)		A(5,8)		A(8,5)		A(6,8)		A(8,6)		A(7,9)			
	A(1,6)		A(6,1)		A(3,5)		A(5,3)		A(3,6)		A(6,3)			
	A(1,9)		A(9,1)		A(2,4)		A(4,2)		A(3,5)		A(5,3)			
	A(9,4)		A(5,7)		A(7,5)		A(5,8)		A(8,5)		A(6,8)			
	A(3,5)		A(5,3)		A(4,6)		A(6,4)		A(4,7)		A(7,4)			
	A(7,5)		A(5,8)		A(8,5)		A(6,8)		A(8,6)		A(7,9)			
	A(1,3)		A(3,1)		A(1,8)		A(8,1)		A(2,7)		A(7,2)			
	A(8,2)		A(3,6)		A(6,3)		A(6,8)		A(8,6)		A(8,2)			

 L13	A(1,5)	A(5,1)	A(2,4)	A(4,2)	A(2,5)	A(2,6)
	A(3,5)	A(5,3)	A(3,9)	A(9,3)	A(4,9)	A(9,4)
 L14	A(1,8)		A(8,1)	A(1,9)		A(2,6)
 L15	A(1,3)	A(3,1)	A(1,9)		A(9,1)	A(2,9)
	A(8,2)	A(2,9)	A(9,2)		A(3,8)	A(8,3)
 L16	A(1,3)	A(3,1)	A(1,7)		A(7,1)	A(3,6)
 L17	A(2,4)	A(4,2)	A(4,6)	A(6,4)	A(4,8)	A(8,4)
 L18	A(1,3)		A(3,1)	A(1,5)		A(5,1)
 L19	A(3,7)			A(7,3)		
 L20	A(3,5)		A(5,3)	A(5,7)		A(7,5)
 L21	A(1,7)	A(7,1)	A(3,5)	A(5,3)	A(3,7)	
	A(7,3)	A(5,9)	A(9,5)	A(7,9)	A(9,7)	
 L22	A(4,8)			A(8,4)		
 L23	A(4,6)		A(6,4)	A(4,9)		A(9,4)
 L24	A(1,3)			A(3,1)		
 L25	A(4,6)	A(6,4)	A(4,7)	A(7,4)	A(7,9)	A(9,7)
 L26	A(1,7)			A(7,1)		

 L30	A(1,6)		A(6,1)		A(2,9)		A(9,2)		A(5,7)		A(7,5)					
	A(5,8)		A(8,5)		A(6,8)		A(8,6)		A(6,9)		A(7,9)		A(9,7)			
 L31	A(1,6)				A(6,1)				A(1,8)				A(8,1)			
 L32	A(2,4)				A(4,2)				A(2,9)				A(9,2)			
 L33	A(4,7)															
 L34	A(2,4)								A(4,2)							
 L35	A(1,6)				A(6,1)				A(1,9)				A(9,1)			
 L36	A(1,5)				A(5,1)				A(1,9)				A(9,1)			
 L37	A(2,4)								A(4,2)							
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 L39	A(1,8)								A(8,1)							

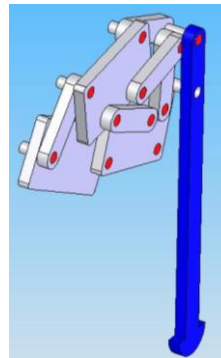
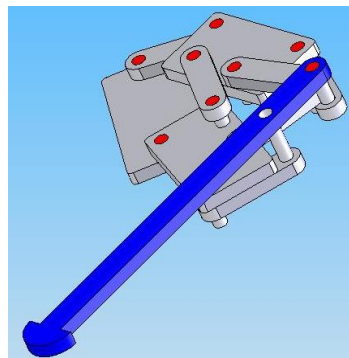
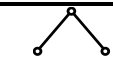
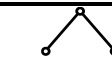
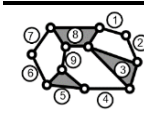
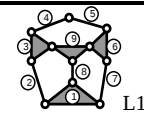
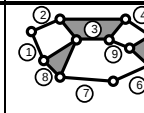
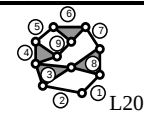
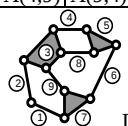
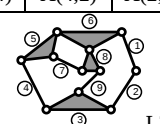
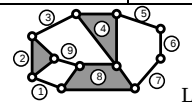
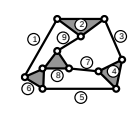
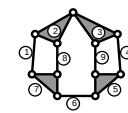
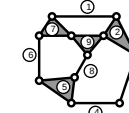
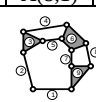
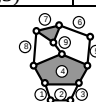
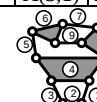
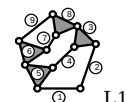
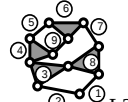
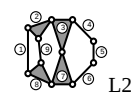
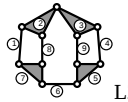
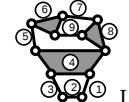
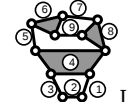
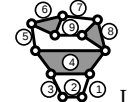


Fig. 12 – Bi-mobile mechanism with 2+2+2+2 inverse modular structure (basis 1 and effector 3) from the linkage 16.

Appendix 2

Base-effector structural solutions from inverse models
with 2+2+4 passive modular groups connection

2+2+4													
													
L10		L15		L17		L20							
A(1,7)	A(7,1)	A(3,5)	A(5,3)	A(3,6)	A(4,9)	A(9,4)	A(1,7)	A(7,1)	A(2,4)	A(4,2)			
													
L23													
A(1,3)	A(3,1)	A(1,9)	A(9,1)	A(2,4)	A(4,2)	A(2,7)	A(7,2)	A(2,9)	A(9,2)	A(3,7)	A(7,3)		
													
L25													
A(1,5)					A(5,1)								
													
L26													
A(4,6)	A(6,4)	A(4,7)	A(7,4)	A(5,7)	A(7,5)	A(5,8)	A(8,5)	A(6,8)	A(8,6)				
													
L27				L29				L30					
A(1,8)	A(8,1)	A(2,6)	A(6,2)	A(1,3)	A(3,1)	A(1,3)	A(3,1)	A(3,8)	A(8,3)				
													
L33				L39				L40					
A(1,7)		A(7,1)		A(1,5)		A(5,1)		A(1,8)		A(8,1)			
2+2+4													
													
L19						L20							
A(1,3)	A(3,1)	A(1,4)	A(4,1)	A(3,5)	A(1,7)		A(7,1)						
													
L28					L29			L40					
A(3,5)	A(5,3)	A(3,6)	A(6,3)	A(4,6)	A(1,6)	A(6,1)	A(1,5)	A(5,1)					
													
L40								L40					

Appendix 3

Base-effector structural solutions from inverse models
with 2+4 +2 passive modular groups connection

2+4+2																							
 L3				 L4				 L5				 L8											
A(2,4)		A(4,2)		A(2,8)		A(8,2)		A(4,9)		A(5,8)		A(2,8)		A(8,2)		A(1,7)		A(7,1)					
 L10				 L12								 L13											
A(3,7)		A(7,3)		A(4,6)		A(6,4)		A(1,6)		A(6,1)		A(2,9)		A(9,2)		A(1,3)		A(3,1)		A(2,8)		A(8,2)	
 L14				 L15				 L16				 L17											
A(1,3)		A(3,1)		A(2,7)				A(1,5)		A(5,1)		A(4,8)		A(3,7)		A(7,3)		A(5,7)		A(7,5)			
 L18				 L19								 L20											
A(2,4)		A(4,2)		A(4,6)		A(6,4)		A(4,6)				A(6,4)		A(4,8)		A(8,4)		A(6,8)		A(8,6)			
 L21				 L22								 L23											
A(4,6)		A(6,4)		A(6,8)		A(8,6)		A(1,6)		A(6,1)		A(3,5)		A(5,3)		A(5,7)				A(7,5)			
 L24				 L25								 L28											
A(2,6)				A(6,2)				A(3,5)				A(5,3)				A(2,4)				A(4,2)			
 L34								 L35															
A(1,6)		A(6,1)		A(4,9)		A(9,4)		A(3,9)								A(9,3)							

2+4+2															
 L3				 L17				 L18				 L20			
A(1,3)		A(3,1)		A(6,8)		A(8,6)		A(3,5)		A(5,3)		A(3,7)		A(7,3)	

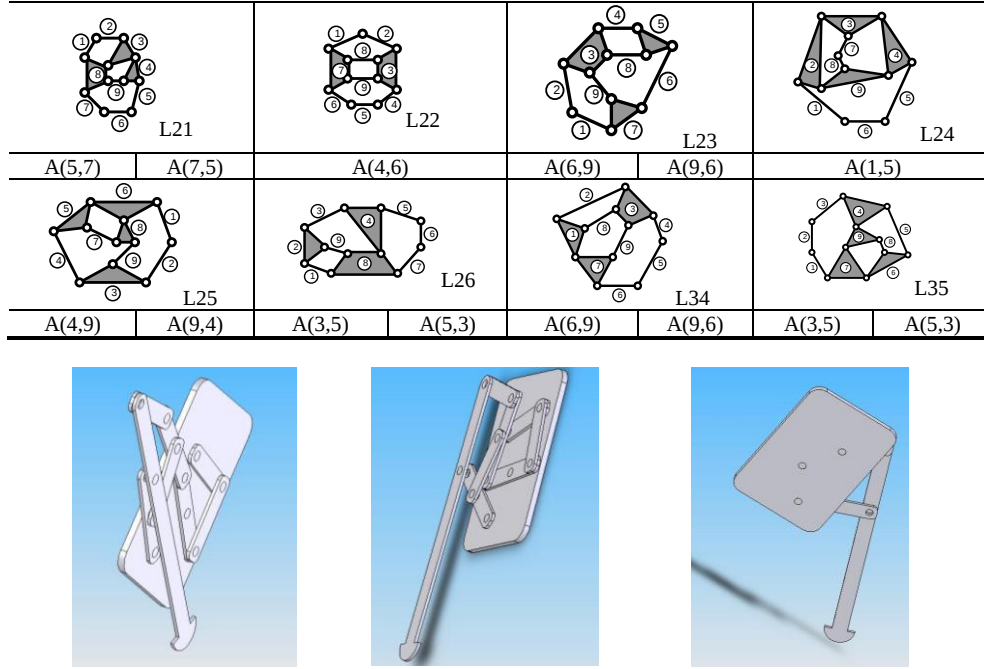
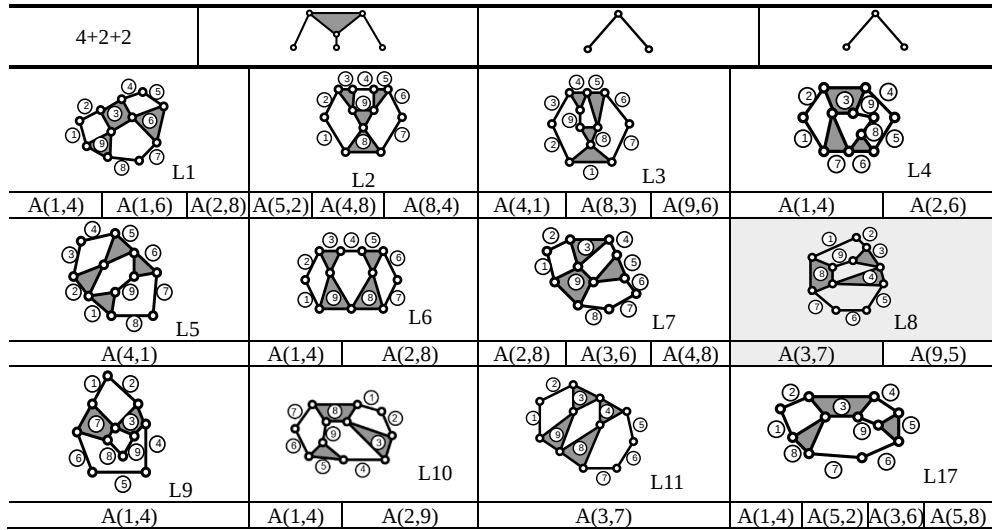
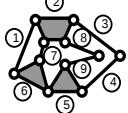
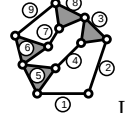
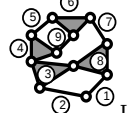
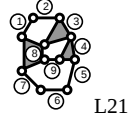
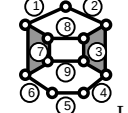
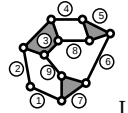
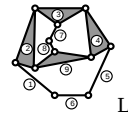
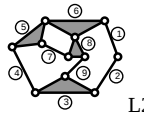
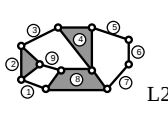
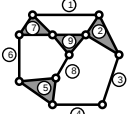
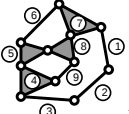
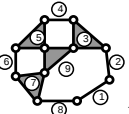
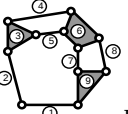
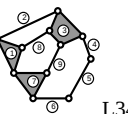
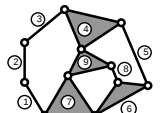
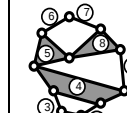
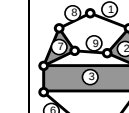
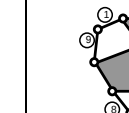
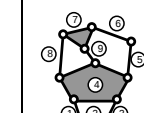
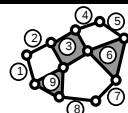
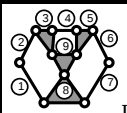
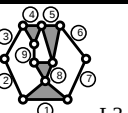
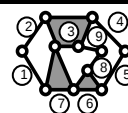
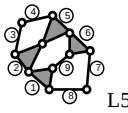
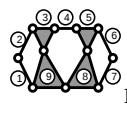
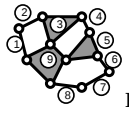
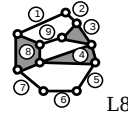
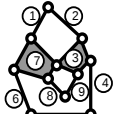
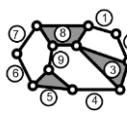
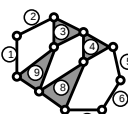
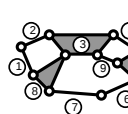
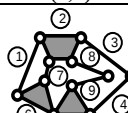
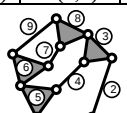
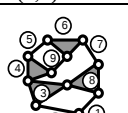
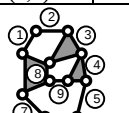


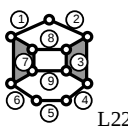
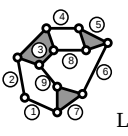
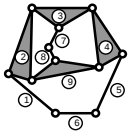
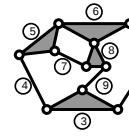
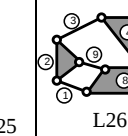
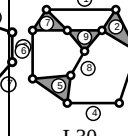
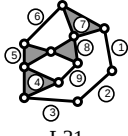
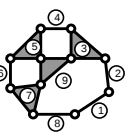
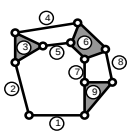
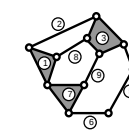
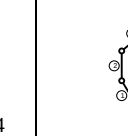
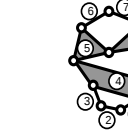
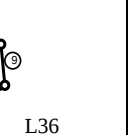
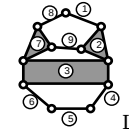
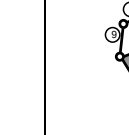
Fig. 13 – Bi-mobile mechanism with 2+4+2 inverse modular structure (basis 2 and effector 8) from L5 linkage.

Appendix 4

Base-effector structural solutions from inverse models
with 4+2+2 passive modular groups connection



														
A(2,5)	A(6,3)	A(6,3)		A(6,3)	A(4,7)		A(2,7)	A(4,7)	A(5,8)					
														
A(3,6)		A(3,6)	A(5,9)	A(9,5)		A(4,1)		A(8,4)	A(5,9)					
														
A(2,7)		A(1,9)	A(2,6)	A(6,2)	A(5,1)		A(5,2)		A(3,7)	A(1,4)				
														
A(8,1)		A(8,1)	A(2,5)	A(2,9)	A(5,2)	A(9,4)	A(1,6)	A(7,2)	A(7,4)	A(7,1)	A(2,8)			
4+2+2														
														
A(4,1)		A(6,1)	A(8,2)		A(2,5)		A(1,4)	A(3,8)		A(6,9)	A(4,1)	A(6,2)		
														
A(1,4)		A(4,1)		A(8,2)		A(8,2)		A(6,3)		A(8,4)	A(7,3)		A(5,9)	
														
A(4,1)		A(4,1)	A(9,2)		A(7,3)		A(4,1)		A(2,5)		A(6,3)		A(8,5)	
														
A(5,2)		A(3,6)		A(3,6)		A(3,6)		A(7,4)		A(7,2)		A(7,4)		A(5,8)

									
A(6,3)	A(6,3)	A(1,4)	A(4,8)	A(9,5)	A(7,2)	A(9,1)			
									
A(1,5)	A(2,5)	A(7,3)	A(4,1)	A(1,8)					
									
A(1,8)	A(5,2)	A(9,2)	A(2,5)	A(4,9)	A(6,1)	A(2,7)	A(4,7)	A(1,7)	A(8,2)

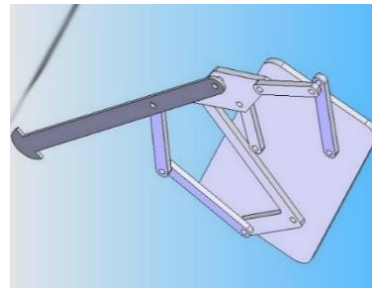
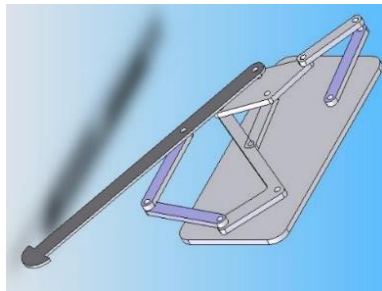
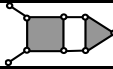
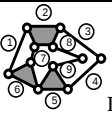
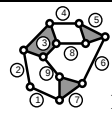
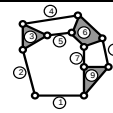
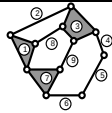
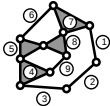
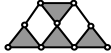
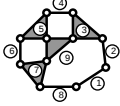
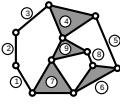
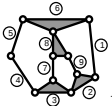
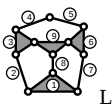
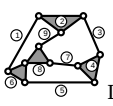
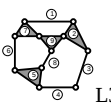
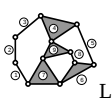
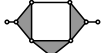
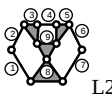
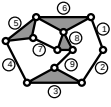
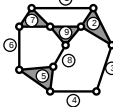
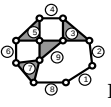
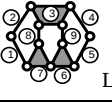
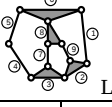
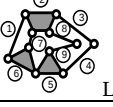
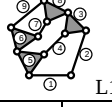


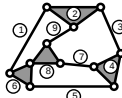
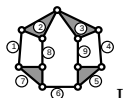
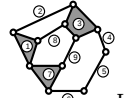
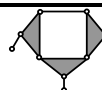
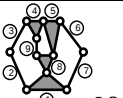
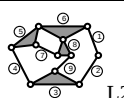
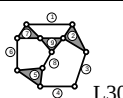
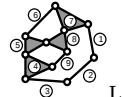
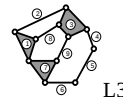
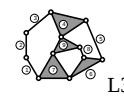
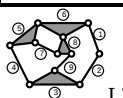
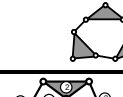
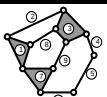
Fig. 14 –Bi-mobile mechanism with 4+2+2 inverse modular structure (basis 3 and effector 7) from L8 linkage.

Appendix 5

Base-effector structural solutions from inverse models
with 2+6 passive modular groups connection

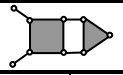
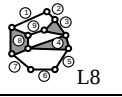
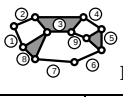
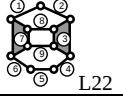
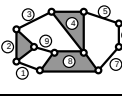
2+6						
						
A(4,9)	A(1,6)	A(6,1)	A(1,3)	A(3,1)	A(5,7)	A(7,5)
2+6						

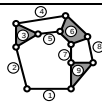
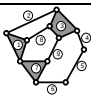
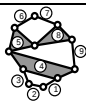
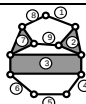
				L31	
		A(1,3)			
2+6					
				L32	
		A(2,8)			
2+6					
				L35	
		A(1,3)		A(3,1)	
2+6					
					
L14		L15		L27	
					
L30		L35			
A(1,5)	A(5,1)	A(3,5)	A(5,3)	A(2,4)	A(4,2)
A(1,3)	A(3,1)		A(2,4)	A(4,2)	A(2,7)
				A(7,2)	
2+6					
					
L2		L25		L30	
					
L32					
A(1,7)		A(1,3)		A(3,1)	
		A(4,9)		A(9,4)	
		A(1,3)		A(3,1)	
2+6					
					
L12		L14		L18	
					
L19					
A(2,4)		A(2,4)		A(3,8)	
		A(4,2)		A(1,6)	
				A(6,1)	

							
A(2,4)		A(5,7)		A(3,5)	A(5,3)		
2+6							
							
A(2,7)	A(7,2)	A(2,4)	A(4,2)	A(3,5)	A(5,3)	A(4,6)	A(6,4)
							
A(2,4)	A(4,2)	A(3,5)	A(5,3)	A(2,4)	A(4,2)		
2+6							
A(2,6)		A(6,2)		A(1,5)			
2+6							
A(4,6)		A(6,4)					

Appendix 6

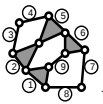
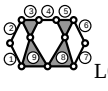
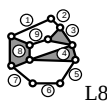
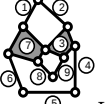
Base-effector structural solutions from inverse models
with 6+2 passive modular groups connection

6+2					
					
A(1,5)	A(2,7)	A(1,5)	A(5,1)	A(1,4)	A(3,7)

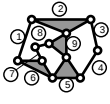
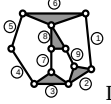
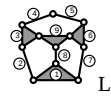
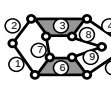
				
L33	L34	L36	L37	
A(3,9)	A(7,4)	A(2,8)	A(8,2)	A(5,9)
				
L38		L39		
A(1,7)	A(7,1)	A(6,1)	A(2,7)	A(7,2)

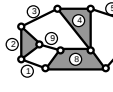
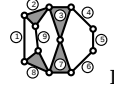
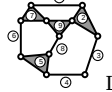
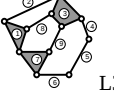
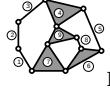
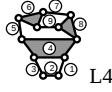
6+2		
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L1		L2		L3		L4			
A(1,7)	A(2,7)	A(4,1)	A(5,1)	A(4,7)	A(9,7)	A(1,5)	A(2,5)	A(4,8)	A(8,4)

								
L5			L6		L8		L9	
A(2,7)	A(7,2)	A(3,9)	A(1,5)	A(2,5)	A(3,6)	A(9,6)	A(1,5)	

										
L10						L11	L12			
A(1,5)	A(2,5)	A(3,6)	A(6,3)	A(4,7)	A(7,4)	A(3,6)	A(1,9)	A(9,1)	A(2,6)	A(6,2)

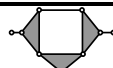
										
L13				L14		L15		L16		
A(1,4)	A(4,1)	A(2,7)	A(7,2)	A(1,7)	A(3,6)	A(2,6)	A(6,2)	A(1,4)	A(4,1)	A(4,9)

							
A(1,5)	A(1,4)	A(1,5)	A(2,5)	A(2,6)	A(7,3)	A(8,3)	A(7,1)

6+2		
-----	---	---

						
L1	L2		L3		L4	
A(7,1)	A(1,4)	A(1,5)	A(7,4)	A(7,9)	A(5,1)	A(5,2)

									
A(9,3)	A(5,1)	A(5,2)	A(2,6)	A(7,3)	A(7,4)	A(1,5)	A(2,5)	A(6,3)	A(6,9)
									
A(5,1)	A(5,1)	A(5,2)	A(1,5)			A(1,5)		A(5,1)	A(5,2)
									
A(4,1)	A(6,2)	A(4,1)	A(5,1)	A(7,1)			A(1,4)		

6+2							
							
A(7,2)	A(2,5)	A(2,9)	A(1,6)	A(1,7)	A(6,3)	A(7,4)	
							
A(4,1)	A(4,7)	A(8,5)	A(2,5)	A(6,3)	A(6,9)	A(5,8)	A(8,5)
							
A(7,4)	A(3,6)	A(3,7)	A(9,1)	A(7,2)	A(6,1)		

6+2						
						
A(5,2)	A(9,2)	A(4,7)	A(1,4)	A(7,4)	A(5,8)	
						
A(3,6)	A(9,6)	A(4,7)	A(3,6)	A(6,3)	A(6,3)	A(7,3)

6+2																							
		L7				L11				L21				L31				L32				L36	
A(6,2)		A(5,1)		A(5,1)		A(1,4)		A(2,6)		A(1,7)													

6+2																							
		L8				L24				L26				L28				L34					
A(5,1)		A(5,2)		A(1,7)		A(7,1)		A(5,1)		A(4,1)		A(6,2)											
				L35								L37								L40			
A(1,4)		A(1,5)		A(3,7)		A(3,8)		A(4,1)		A(1,7)													

6+2																							
		L8				L22				L26				L28				L34					
A(7,2)		A(4,1)		A(5,2)		A(7,3)		A(6,2)		A(6,3)		A(4,7)											
				L35								L39								L40			
A(3,6)				A(1,6)				A(1,6)															

6+2																							
		L26				L28				L34				L35				L40					
A(2,5)		A(2,6)		A(3,6)		A(6,3)		A(6,1)															

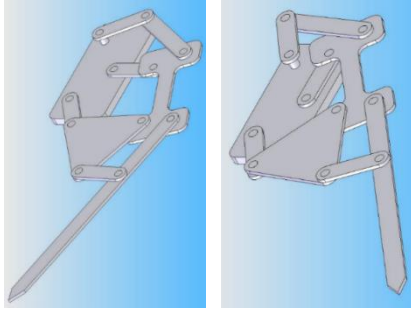


Fig. 15 – Bi-mobile mechanism with 6+2 inverse modular structure (basis 5 and effector 1) from L10 linkage.

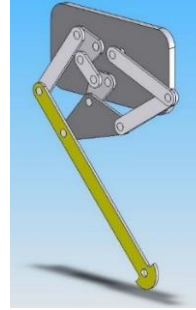


Fig. 16 – Bi-mobile mechanism with 6+2 inverse modular structure (basis 5 and effector 2) from L3 linkage.

Appendix 7

Base-effector structural solutions from inverse models
with 4+4 passive modular groups connection

4+4				
 L10	 L17	 L20	 L23	 L29
A(2,7)	A(2,7)	A(1,4)	A(5,2)	A(5,2)
 L30	 L33	 L39	 L40	
A(7,3)	A(6,1)	A(2,5)	A(2,8)	
4+4				
 L10	 L17	 L20	 L23	
A(7,2)	A(7,2)	A(4,1)	A(2,5)	
 L25		 L29		
A(1,7)	A(7,1)	A(2,5)		
 L30	 L33	 L39	 L40	
A(3,7)	A(1,6)	A(5,2)	A(8,2)	

4+4		
A(2,7)	A(2,6)	A(2,5)

4+4		
A(7,2)	A(6,2)	A(5,2)

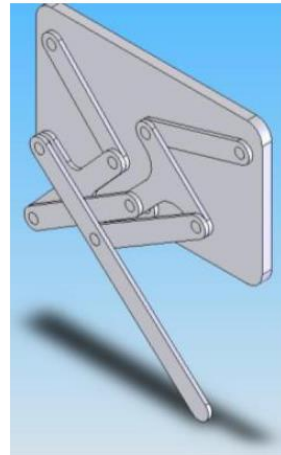
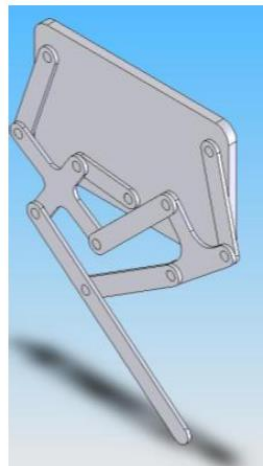


Fig. 17 – Bi-mobile mechanism with 4+4 inverse modular structure (basis 5 and effector 2) from L23 linkage.