VIBRATION ANALYSIS OF THE STRUCTURES WITH IDENTICAL PARTS USING FINITE ELEMENT METHOD

RADU POPESCU , CRISTINA MICLEA, LUCIAN GABRIEL DOMINTE, OMAR ABDULAH OMAR SHRRAT, CALIN ITU, SORIN VLASE

Abstract. In many engineering structures, and especially in the field of civil engineering, there are systems that have identical parts or that have certain symmetries. These structural properties give the possibility to ease the design and calculation in such circumstances. Thus, the analysis time can be reduced and the quick estimation of the response offered by the structure in different operating conditions can be reduced. As a result, the cost price for building the structure decreases accordingly. The symmetries of the structures and the existing identical parts also allow an easier analysis in the case of the study of vibrations. In the work, specific vibration properties of a structure containing identical parts are highlighted. An example for a real-life structure will argue the presented results.

Key words: mechanical structure, eigenfrequency, eigenmode, vibration, identical parts, symmetry.

1. INTRODUCTION

Symmetries appear frequently in many of the structures, machines and devices created by man, for various reasons. In engineering, examples of systems that have different types of symmetries or systems that contain identical parts represent a common situation. Especially in civil engineering, the different types of constructions are characterized by symmetries or identical parts. They are something common in modern constructions, especially in public structures such as stadiums, roofs of exhibition grounds, sports halls or exhibitions, administrative buildings, department stores, warehouses, railway stations, airports, etc. The use of projects that contain identical parts brings a series of logistical or economic advantages. Design, analysis and construction time can be reduced, sometimes significantly. Obviously, building a building becomes easier if the number of elements used is reduced. And from an engineering point of view, the static and dynamic analysis of such a structure becomes easier [1–4].

The advantages of using these symmetries have been observed and used empirically for a long time by users of the finite element method (FEM). First of all,

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the amount of useful information for applications is less. This determines the reduction of design effort and time, and the reduction of the volume of calculations significantly facilitates a dynamic analysis of the structure. The specialized literature has signaled and highlighted the importance of regularities and symmetries [5–6]. It can be observed that the research in this field has focused on the mathematical formalism and the simplifications that can be made if these symmetries are considered. Obviously, the result is obtaining with less effort the equations of motion of the system. In the aeronautical industry, the advantage of using symmetries in the description of mechanical systems that can be used in the design stage to ease this process and reduce costs have been observed for a long time. The generation of a single repetitive structure allows to facilitate the approach of the model as a whole [7]. The additional properties arising from the use of these symmetries allow the use of simplified methods for solving the equations of motion [8].

The modern technical solutions currently offered by civil engineering for the protection of large spaces such as exhibition halls, stadiums, concert halls or different types of storage spaces are layered spatial networks (in two or three layers). Such a structure presents different types of symmetries, both locally and globally, at the level of the entire structure. Using group theory, an analysis of such a structure is carried out in [9] for the study of the response to vibrations. It is observed that a building with symmetries tends to be more stable to various anthropic activities or in the case of natural phenomena. Different aspects of the problem are presented in [10–11] and some results concerning different engineering problems using symmetry are described in [12–16].

In architecture and building practice, the main types of known symmetries are used, such as bilateral symmetry, rotational symmetry, chiral symmetry and axial symmetry [17–19]. In this study, the case of a rotational symmetry, which can be frequently encountered in the practice of building construction, will be analysed. The paper will present the properties of the equations of motion of such systems. They can have favourable effects on the effort of dynamic and vibration analysis of such structures. An application for the calculation of a cooler support structure will illustrate the results obtained.

2. METHODS

The equations of motion for a mechanical system with linear elastic elements and viscous damping generally have the established form [20–28]:

$$[M]\{\ddot{\Delta}\}+[C]\{\dot{\Delta}\}+[K]\{\Delta\}=\{F\}\,. \tag{1}$$

If we consider an elastic mechanical system made up of two identical parts connected in a similar way to a third linear elastic mechanical system, the equations that describe the evolution of every part, taken individually (Fig. 1) are:

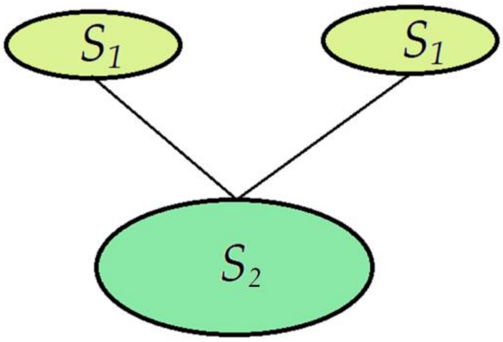


Fig. 1 – Elastic system containing two identical parts.

$$[M_1]\{\ddot{\Delta}_{11}\}+[C_1]\{\dot{\Delta}_{11}\}+[K_1]\{\Delta_{11}\}=\{F_1\}, \tag{2}$$

$$[M_1]\{\ddot{\Delta}_{12}\}+[C_1]\{\dot{\Delta}_{12}\}+[K_1]\{\Delta_{12}\}=\{F_1\}, \tag{3}$$

$$[M_2]\{\ddot{\Delta}_2\}+[C_2]\{\dot{\Delta}_2\}+[K_2]\{\Delta_2\}=\{F_2\}. \tag{4}$$

If we now consider the entire system as a whole, the motion equations that describe its evolution, considering the connections between these systems defined by the M_{12}, C_{12} and K_{12} matrices are given by the equations:

$$\begin{bmatrix} M_1 & 0 & M_{12} \\ 0 & M_1 & M_{12} \\ M_{12}^T & M_{12}^T & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{\Delta}_{11} \\ \ddot{\Delta}_{12} \\ \ddot{\Delta}_2 \end{Bmatrix}+\begin{bmatrix} C_1 & 0 & C_{12} \\ 0 & C_1 & C_{12} \\ C_{12}^T & C_{12}^T & C_2 \end{bmatrix}\begin{Bmatrix} \Delta_{11} \\ \Delta_{12} \\ \Delta_2 \end{Bmatrix}+ \\ +\begin{bmatrix} K_1 & 0 & K_{12} \\ 0 & K_1 & K_{12} \\ K_{12}^T & K_{12}^T & K_2 \end{bmatrix}\begin{Bmatrix} \Delta_{11} \\ \Delta_{12} \\ \Delta_2 \end{Bmatrix}=\begin{Bmatrix} F_1 \\ F_1 \\ F_2 \end{Bmatrix}, \tag{5}$$

where: M_1, K_1, C_1 are respectivelly the inertial, stifness and damping matrices for the subsystems S_1; M_2, K_2, C_2 are respectivelly the inertial, stifness and damping matrices for the subsystems S_2 and M_{12}, K_{12}, C_{12} are respectivelly the coupling inertial, stifness and damping matrices between the two identical parts S_1 and the system S_2; Δ_{1i} – the vector of independent coordinates for subsystem S_i; Δ_2 – the vector of independent coordinates for the subsystem S_2.

Such types of systems were studied in literature in the last periods [29–33].

Let's us consider the system of differential equations of undamped free vibrations of the system. Equation (5) becomes:

$$\begin{bmatrix} M_{11} & 0 & M_{12} \\ 0 & M_{11} & M_{12} \\ M_{12}^T & M_{12}^T & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{\Delta}_{11} \\ \ddot{\Delta}_{12} \\ \ddot{\Delta}_{2} \end{Bmatrix} + \begin{bmatrix} K_{11} & 0 & K_{12} \\ 0 & K_{11} & K_{12} \\ K_{12}^T & K_{12}^T & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta_{11} \\ \Delta_{12} \\ \Delta_{2} \end{Bmatrix} = \{0\}. \tag{6}$$

The characteristic equation for Eq.(6) is:

$$\begin{vmatrix} K_{11} - p^2 M_{11} & 0 & K_{12} - p^2 M_{12} \\ 0 & K_{11} - p^2 M_{11} & K_{12} - p^2 M_{12} \\ K_{12}^T - p^2 M_{12}^T & K_{12}^T - p^2 M_{12}^T & K_{22} - p^2 M_{22} \end{vmatrix} = 0. \tag{7}$$

For a substructure S_1, the system of differential equations of undamped free vibrations is:

$$[M_1]\{\ddot{\Delta}_{1i}\} + [K_1]\{\Delta_{1i}\} = 0, \quad i = 1,2, \tag{8}$$

and the characteristic equation becomes:

$$\det\left([K_{11}] - p^2 [M_{11}]\right) = 0. \tag{9}$$

Some fundamental results are presented in [35–37]. The main result that will be used in the case of the work is:

THEOREM: *If we have a square matrix with complex coefficients, of the form*:

$$M = \begin{pmatrix} A & Z & B \\ Z & A & B \\ L & L & C \end{pmatrix}. \tag{10}$$

It is proved that det(*M*) is divisible by det(*A*). A demonstration can be found in [38]. This result allows formulating the following properties for the vibrations of a system with two identical parts:

P1. The eigenvalues for the identical parts are also the eigenvalues for the whole structure.

P2. For the eigenvalues considered in P1 the eigenmodes are of the form:

$$\Phi = \begin{Bmatrix} \Phi_1 \\ -\Phi_1 \\ 0 \end{Bmatrix} \tag{11}$$

(the components of the eigenmodes corresponding to the identical parts are antisymmetric).

P3. For the other eigenvalues, the eigenvectors are of the form:

$$\Phi = \begin{Bmatrix} \Phi_1 \\ \Phi_1 \\ \Phi_2 \end{Bmatrix} \tag{12}$$

(for the other eigenvalues of the system the components of the eigenmodes corresponding to the identical parts are identical).

In the following, we will show that these types of properties can be extended to systems with more 2 identical parts. Consider a principal system being in liaison with n identical system. The symmetry of the main system S_0 it allows the other S_1 systems to connect to it in the same way. So the presented properties will be expanded for these types of systems (Fig. 2).

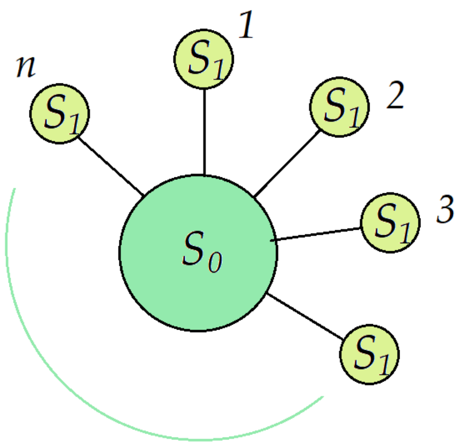


Fig. 2 – A sytem with n identical parts.

The motion equations fo a system S_1 taken individually are:

$$[M_{11}]\{\ddot{\Delta}_{1i}\} + [K_{11}]\{\Delta_{1i}\} = \{0\}, \tag{13}$$

and for the system S_2 taken individually:

$$[M_{22}]\{\ddot{\Delta}_2\} + [K_{22}]\{\Delta_2\} = \{0\}. \tag{14}$$

The modeling of the two systems was done in the paper using the Finite Element Method (FEM). The program used for the calculation was ABAQUS. If we consider the structure formed by two such substructures and if we neglect the structural damping and other types of proportional damping that may occur, then the equations of motion of the free vibrations are of the form of Eq. (1) where the matrices that define the system are:

$$[M]=\begin{bmatrix} M_{11} & & & & M_{12} \\ & M_{11} & & 0 & M_{12} \\ & & \ddots & & \vdots \\ & 0 & & M_{11} & M_{12} \\ M_{12}^T & M_{12}^T & \dots & M_{12}^T & M_{22} \end{bmatrix}, \tag{15}$$

$$[K]=\begin{bmatrix} K_{11} & & & & K_{12} \\ & K_{11} & & 0 & K_{12} \\ & & \ddots & & \vdots \\ & 0 & & K_{11} & K_{12} \\ K_{12}^T & K_{12}^T & \dots & K_{12}^T & K_{22} \end{bmatrix}, \tag{16}$$

$$[C]=\begin{bmatrix} C_{11} & & & & C_{12} \\ & C_{11} & & 0 & C_{12} \\ & & \ddots & & \vdots \\ & 0 & & C_{11} & C_{12} \\ C_{12}^T & C_{12}^T & \dots & C_{12}^T & C_{22} \end{bmatrix}. \tag{17}$$

The condition to have an harmonic solution offers us the linear system:

$$\left\{\begin{bmatrix} K_{11} & & & & K_{12} \\ & K_{11} & & 0 & K_{12} \\ & & \ddots & & \vdots \\ & 0 & & K_{11} & K_{12} \\ K_{12}^T & K_{12}^T & \dots & K_{12}^T & K_{22} \end{bmatrix} - \right. \\ \left. -p2\begin{bmatrix} M_{11} & & & & M_{12} \\ & M_{11} & & 0 & M_{12} \\ & & \ddots & & \vdots \\ & 0 & & M_{11} & M_{12} \\ M_{12}^T & M_{12}^T & \dots & M_{12}^T & M_{22} \end{bmatrix}\right\}\begin{Bmatrix} \Phi_{11} \\ \Phi_{12} \\ \vdots \\ \Phi_{1n} \\ \Phi_2 \end{Bmatrix}=\{0\}, \tag{18}$$

or, developing:

$$
\begin{aligned}
&\left([K_{11}]-p^2[M_{11}]\right)\{\Phi_{11}\}+\left([K_{12}]-p^2[M_{12}]\right)\{\Phi_2\}=0,\\
&\left([K_{11}]-p^2[M_{11}]\right)\{\Phi_{12}\}+\left([K_{12}]-p^2[M_{12}]\right)\{\Phi_2\}=0,\\
&\qquad\qquad\ldots.\\
&\left([K_{11}]-p^2[M_{11}]\right)\{\Phi_{1n}\}+\left([K_{12}]-p^2[M_{12}]\right)\{\Phi_2\}=0,\\
&\left([K_{12}]^T-p^2[M_{12}]^T\right)\left(\{\Phi_{11}\}+\{\Phi_{12}\}+\ldots+\{\Phi_{1n}\}\right)+\\
&+\left([K_{22}]-p^2[M_{22}]\right)\{\Phi_2\}=0.
\end{aligned}
\tag{19}
$$

Will be denote:

$$
\begin{gathered}
[A_{11}]=[K_{11}]-p^2[M_{11}],\quad [A_{22}]=[K_{22}]-p^2[M_{22}],\\
[A_{12}]=[K_{12}]-p^2[M_{12}].
\end{gathered}
\tag{20}
$$

Using these notations, it is possible to write the Eq. (18) under the form:

$$
\begin{aligned}
&[A_{11}]\{\Phi_{11}\}+[A_{12}]\{\Phi_2\}=0,\\
&[A_{11}]\{\Phi_{12}\}+[A_{12}]\{\Phi_2\}=0,\\
&\qquad\ldots\ldots\ldots\ldots\\
&[A_{11}]\{\Phi_{1n}\}+[A_{12}]\{\Phi_2\}=0,\\
&[A_{12}]^T\left(\{\Phi_{11}\}+\{\Phi_{12}\}+\ldots+\{\Phi_{1n}\}\right)+[A_{22}]\{\Phi_2\}=0.
\end{aligned}
\tag{21}
$$

THEOREM 1. *If an eigenvalue for the system is also an eigenvalue for* S_1, *then the eigenvector associated with this eigenvalue is of the form*:

$$
\{\Phi\}=\begin{Bmatrix}\Phi_{11}\\ \Phi_{12}\\ \vdots\\ \Phi_{1n}\\ 0\end{Bmatrix},
\tag{22}
$$

with:

$$
\{\Phi_{11}\}+\{\Phi_{12}\}+\ldots+\{\Phi_{1n}\}=0\,.
\tag{23}
$$

Proof: If $\det[A_{11}]=0$ it exists $\{\Phi_{11}\}\neq 0$ so:

$$
[A_{11}]\{\Phi_{11}\}=0\,.
\tag{24}
$$

In this case the first relation from Eq. (21) becomes:

$$[A_{12}]\{\Phi_2\} = 0\,, \tag{25}$$

from where, generally:

$$\{\Phi_2\} = 0\,. \tag{26}$$

Introducing in the last relation of Eq. (21) it obtains:

$$\{\Phi_{11}\} + \{\Phi_{12}\} + \ldots + \{\Phi_{1n}\} = 0\,. \tag{27}$$

Thus, the eigenmode of vibration will have the form:

$$\{\Phi\} = \begin{Bmatrix} \Phi_{11} \\ \Phi_{12} \\ \vdots \\ \Phi_{1n} \\ 0 \end{Bmatrix}, \quad \text{with } \{\Phi_{11}\} + \{\Phi_{12}\} + \ldots + \{\Phi_{1n}\} = 0.$$

THEOREM 2. *For the other eigenpulsations the eigenvectors have the form*:

$$\{\Phi\} = \begin{Bmatrix} \Phi_1 \\ \Phi_1 \\ \vdots \\ \Phi_1 \\ \Phi_2 \end{Bmatrix}. \tag{28}$$

Proof: Subtracting the first two relations of Eq. (21) yields:

$$[A_{11}]\left(\{\Phi_{11}\} - \{\Phi_{12}\}\right) = 0\,, \tag{29}$$

whence, since $\det[A_{11}] \neq 0$, it results:

$$\{\Phi_{11}\} - \{\Phi_{12}\} = 0\,, \tag{30}$$

so:

$$\{\Phi_{11}\} = \{\Phi_{12}\} = \{\Phi_1\}\,. \tag{31}$$

The reasoning can be repeated for the other equations as well. It results immediately:

$$\{\Phi_{11}\} = \{\Phi_{12}\} = \ldots = \{\Phi_{1n}\}\,. \tag{32}$$

If we denoted all these equal vectors with $\{\Phi_1\}$ it results the corresponding eigenvector in the form offered by Eq. (28).

3. RESULTS

The previously highlighted properties are exemplified when calculating a support structure for an industrial cooler. The structure consists of a set of metal trusses, of standardized dimensions. To make the structure, 4 types of trusses are used, stiffened between them by welding. Within Fig. 3 shows the structure, made up of 56 bars joined together by 24 nodes. There are two symmetry planes of the structure, *y*O*z* and *x*O*y*.

The structure is composed of a lower sole and an upper sole realized of hot-rolled square pipe with a section of 30×2 mm, with uprights made of hot-rolled square pipe with a section of 20×2 mm. The diagonals are obtained using hot-rolled square pipe with a section of 20×2 mm, and the uprights at the ends, which also act as pillars, are made of hot-rolled square pipe with a section of 40×2.5mm. Finally, the sleepers are made of laminated profiles with the U65 section. All the described elements were made of S235JR steel. The assembly and stiffening was carried out by welding on the entire contact surface of the parts, with a thickness of $0.7\times T_{\min}$. For this structure, the natural frequencies and proper vibration modes were calculated. Four cases will be studied. The entire structure, then two substructures obtained by sectioning with a plane of symmetry of the assembly and finally a quarter of the structure that can be obtained by sectioning with a plane of symmetry of one of the two considered substructures (Figs. 5, 6).

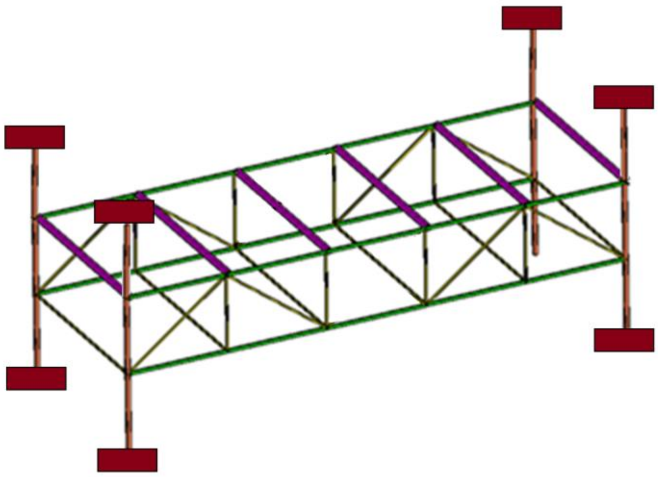


Fig. 3 – Metallic structure.

The dimensions of the structure are presented in Fig. 4.

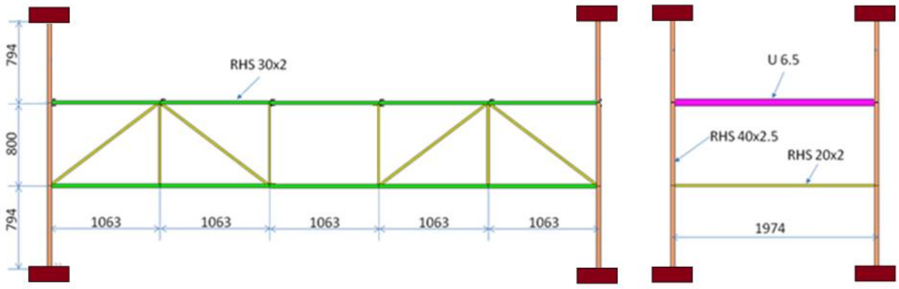
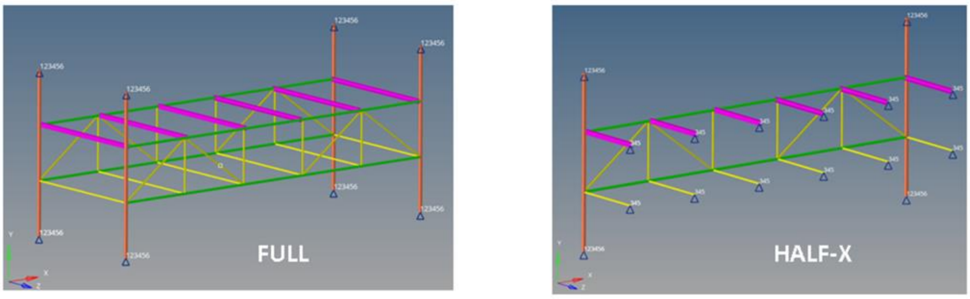
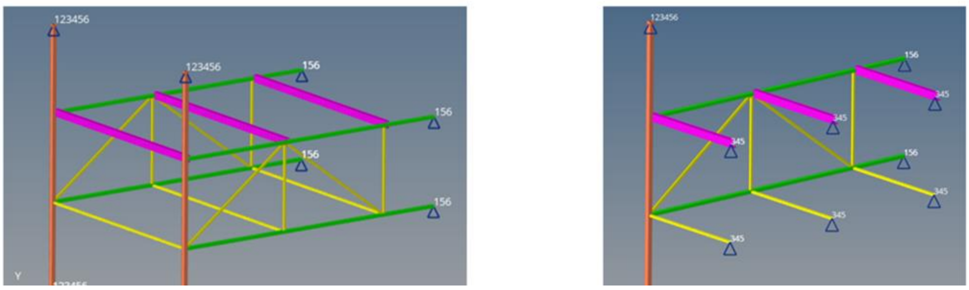


Fig. 4 – Dimensions of the structure.



a. b.

Fig. 5 – a) Full structure; b) symmetry plane is O*yz*.



a. b.

Fig. 6 – a) Symmetry plane is O*xy*; b) quarter.

The eigenfrequencies are determined and presented, for all four cases, in Table 1.

Table 1

The computed eigenfrequencies

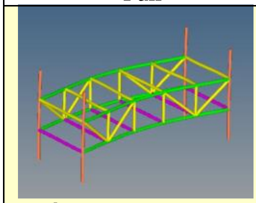
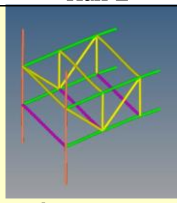
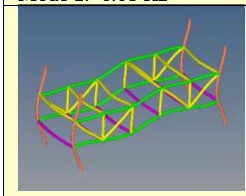
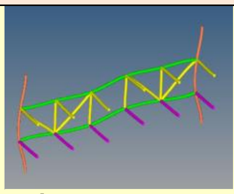
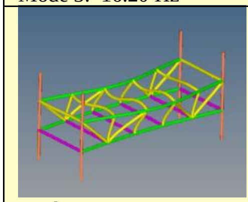
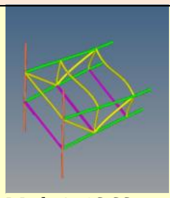
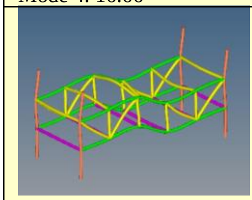
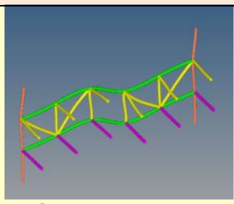
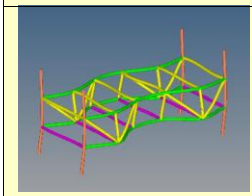
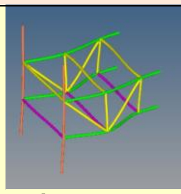
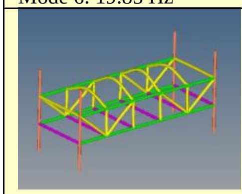
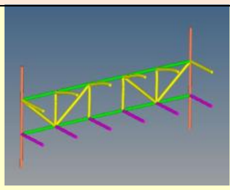
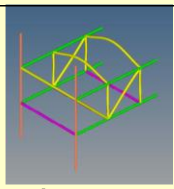
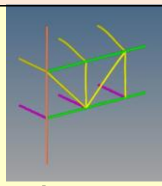
FULL		HALF_X		HALF_Z		QUARTER	
Mode	Eigen frequency [Hz]	Mode	Eigen-frequency [Hz]	Mode	Eigen-frequency [Hz]	Mode	Eigen-frequency [Hz]
1	6.08			1	6.08		
2	12.64						
3	16.20	1	16.20				
4	16.66			2	16.66		
5	19.13	2	19.13				
6	19.85			3	19.85		
7	22.44						
8	23.66						
9	26.17						
10	26.85						
11	27.50	3	27.50	4	27.50	1	27.50
12	29.25	4	29.25				
13	29.89	5	29.89	5	29.89	2	29.89
14	31.02	6	31.02				
15	31.15	7	31.15	6	31.15	3	31.15
16	31.73	8	31.73				
17	32.02			7	32.02		
18	32.40	9	32.40	8	32.40	4	32.40
19	32.93	10	32.93				
20	33.09	11	33.09	9	33.09	5	33.09
21	34.20	12	34.20				
22	34.28	13	34.28	10	34.28	6	34.28
23	34.28	14	34.28	11	34.38		
24	34.38	15	34.38			7	34.38
25	34.61						
26	35.47			12	35.47		
27	35.57	16	35.57				
28	36.43	17	36.43	13	36.43	8	36.43
29	37.03	18	37.03	14	37.03	9	37.03
30	38.36	19	38.36				
31	40.11	20	40.11	15	40.11	10	40.11

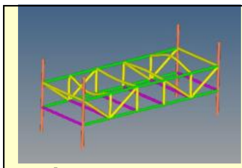
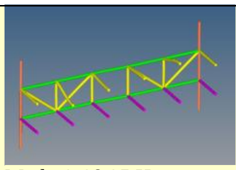
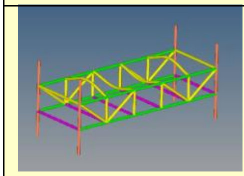
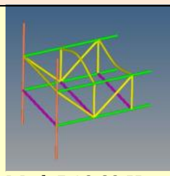
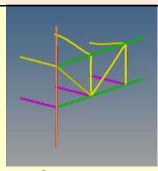
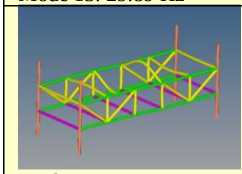
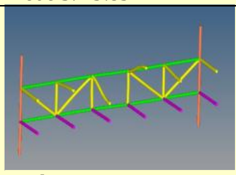
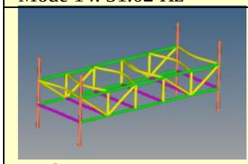
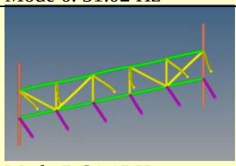
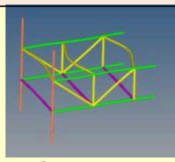
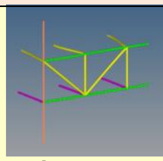
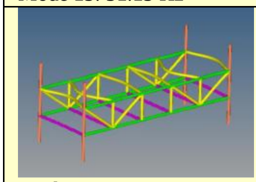
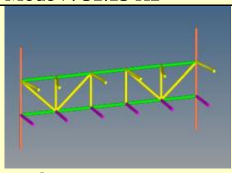
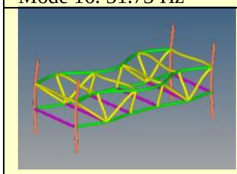
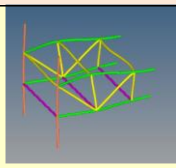
32	43.65						
33	46.97		46.97	16	46.97	11	46.97
34	47.22						
35	53.39						
36	54.58			17	54.58		
37	56.79		56.79	18	56.79	12	56.79
38	56.80						
39	61.71						
40	65.37		65.37	19	65.37	13	65.37
41	66.49				66.49		
42	68.22						
43	69.61						
44	70.35		70.35	20	70.35	14	70.35
45	70.93						
46	71.07						
47	71.87		71.87		71.87	15	71.87
48	72.27						
49	72.92						
50	73.72		73.72		73.72	16	73.72

The example presented illustrates the previously demonstrated properties. We have the natural frequencies calculated for the entire structure. Then, if only one half structure is considered, symmetrical to the *y*O*z* plane (called Half X), all the 20 calculated eigenfrequencies are also the natural frequencies of the whole structure. The property remains valid if a second symmetry is considered, for the second half-structure (called Half Z), with the plane of symmetry *x*O*y*. Finally, if we now consider a quarter structure, that is, the structure consists of four identical parts, it is found that the own frequencies of the quarter structure are found among the frequencies of the Half X, Half Z and Full structures. If one analyzes the representation of the natural modes of vibration, it can be seen that the previously stated properties are respected. The presented example validates the theoretical results. Based on the results of the natural frequencies calculated and presented in Table 1, a representation of the natural modes of vibration for the symmetrical modes was made, in Table 2.

Table 2

The Eigenmodes of vibration

Type of symmetry			
Full	Half X	Half Z	Quater
 Mode 1: 6.08 Hz		 Mode1:6.08 Hz	
 Mode 3: 16.20 Hz	 Mode 1: 16.20 Hz		
 Mode 4: 16.66		 Mode 2: 16.66	
 Mode 5: 19.13 Hz	 Mode 2: 19.13 Hz		
 Mode 6: 19.85 Hz		 Mode 3: 19.85	
 Mode 11: 27.50 Hz	 Mode 3: 27.50 Hz	 Mode 4: 27.50 Hz	 Mode 1:27.50 Hz

 Mode 12: 29.25 Hz	 Mode 4: 29.25 Hz		
 Mode 13: 29.89 Hz	 Mode 5: 29.89 Hz	 Mode5:29.89 Hz	 Mode 2:29.89 Hz
 Mode 14: 31.02 Hz	 Mode 6: 31.02 Hz		
 Mode 15: 31.15 Hz	 Mode 7: 31.15 Hz	 Mode 6: 31.15 Hz	 Mode 3:31.15 Hz
 Mode 16: 31.73 Hz	 Mode 8: 31.73 Hz		
 Mode 17: 32.02 Hz		 Mode 7: 32.02 Hz	

4. DISCUSSION

The paper used FEM to determine the spectrum of natural frequencies and the representation of natural modes of vibration for a structure made up of trusses connected by welding. The studied structure is used to fix a cooling installation on a building. The obtained truss system consists of four types of bars rigidly connected to each other by welding. For this system, the natural frequencies and modes of vibration were calculated. The paper is added to other contributions of the group of authors regarding the vibrations of mechanical systems with symmetries or with identical parts and comes to complete previously obtained results, in order to provide a more suggestive picture of this type of problem.

The studied problem presents two planes of symmetry. Within the obtained results, it can easily be seen that all the eigenfrecquenciesof the substructures defined by the symmetry properties can be found among the eigenfrequencies of the whole structure.

Thus it is possible to solve simpler problems, to determine the natural frequencies on the substructures. These can then be eliminated from the equations written for the entire structure. In this way, the size of the system for which the natural frequencies must be determined decreases and therefore the calculation effort can be significantly reduced. Obviously, the qualitative representations are also a factor that we have to take into account, in this way we can have a physical image of the vibration behavior of the structure.

The obtained results can be developed for such systems in the case of forced vibrations, where for systems with symmetries certain terms cancel, so symmetry could be an advantage in the case of extreme phenomena, such as, for example, earthquakes. A study of the influence of different constructive solutions on the reduction of vibrations in the event of an earthquake could be an interesting development.

5. CONCLUSIONS

Considering symmetries can offer advantages on several levels. In the case of the construction of a building, it is obvious that logistics expenses decrease. But they also offer advantages in the field of engineering calculations. For such a system it is possible to determine in a first phase the natural frequencies of the substructures defined by the symmetries. Then they can be eliminated from the equations written for the entire structure. In this way, the size of the analyzed system can be reduced. Obviously, this offers calculation advantages, especially in the case of FEM use, where the number of degrees of freedom is large and the calculation effort substantial.

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