UNCERTAINTY PROPAGATION FOR WIND TUNNEL DATA POST-PROCESSING: APPLICATION TO A TEST MODEL

THEODOR-ANDREI DRAGHICI*[1,2], DANIEL-EUGENIU CRUNTEANU[2], IONUT BUNESCU[1,2], MIHAITA-GILBERT STOICAN[1,2], MIHAI-VLADUT HOTHAZIE[1,2]

Abstract. Reliable wind tunnel data reduction requires a clear quantification of the uncertainty associated with each derived flow parameter and aerodynamic coefficient. In a typical post-processing workflow, the primary measurements, such as total pressure, static pressure, total temperature, model incidence, balance loads and pressure-tap readings, are successively used to calculate other variables like Mach number, static temperature, density, speed of sound, airspeed, dynamic pressure, viscosity, Reynolds number and aerodynamic coefficients. This paper presents a structured methodology for estimating the standard uncertainty of these quantities directly within the data-reduction methodology, using the chain (sensitivity) method, i.e. the first-order law of propagation of uncertainty. Rather than propagating the uncertainty of each intermediate quantity as if it were an independent input, every derived quantity is expressed as a function of the independent primary measurements and differentiated through the data-reduction equations by the chain rule, so that the correlations introduced by shared inputs are handled consistently. The procedure is applied point-by-point to a reference wind tunnel test campaign and demonstrated across four flow regimes: subsonic (M = 0.4), transonic (M = 1.05) and supersonic (M = 2.0 and M = 3.5). The resulting framework provides a traceable, reproducible procedure for embedding uncertainty estimation into wind tunnel data processing and can serve as a reference for similar experimental campaigns.

Key words: Wind tunnel testing, uncertainty propagation, chain method, data post-processing, standard uncertainty, aerodynamic coefficients, flow parameters.

1. INTRODUCTION

Wind tunnel testing remains a cornerstone of aerodynamic development and validation. In spite of the steady progress of computational methods, experimental data is still indispensable for validating numerical models, certifying facilities and supporting design decisions [1]. The value of these data, however, does not lie in

[1]Corresponding author. INCAS – National Institute for Aerospace Research "Elie Carafoli", B-dul Iuliu Maniu 220, 061126 Bucharest, Romania; e-mail: draghici.andrei@incas.ro
[2]Faculty of Aerospace Engineering, National University of Science and Technology POLITEHNICA Bucharest (UNSTPB), Splaiul Independenței 313, 060042 Bucharest, Romania.

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the measured numbers alone. A result reported without an estimate of its uncertainty cannot be compared with a simulation, with a previous campaign, or with a requirement in any rigorous way. A credible test therefore delivers two relevant quantities for every output: a best estimate and the standard uncertainty that bounds it.

The conceptual framework for expressing measurement uncertainty is now well established. The general approach was formalized by the *Guide to the Expression of Uncertainty in Measurement* (GUM) [4] and was translated to engineering practice by the foundational work of Kline and McClintock [7], Moffat [6] and Coleman and Steele [5], with parallel guidance from the NIST [8] and ASME [10, 11] communities. For wind tunnel testing specifically, the methodology was consolidated in AGARD Advisory Report AR-304 [2] and in the AIAA standard S-071A and its supplement [3, 9]. These documents define error sources, sensitivity coefficients and coverage intervals, and they prescribe how the uncertainty of a measured result should be assembled from the uncertainties of its inputs.

In day-to-day post-processing, however, two practical gaps persist. First, uncertainty is frequently reported only as the run-to-run repeatability of a few nominally identical tests, or it is computed once for a small set of representative points, instead of being evaluated continuously and point-by-point along with the reduced data. Second, the data-reduction chain is strongly coupled, meaning that a handful of primary measurements is transformed through a sequence of equations in which a few intermediate quantities (notably, in this paper, the Mach number M and the static temperature T) are reused by many downstream outputs. Propagating the uncertainty of each intermediate quantity as if it were an independent input is convenient, but it silently mishandles the correlations introduced by these shared inputs and can bias the result.

The objective of this paper is to present a transparent and reproducible chain-rule methodology that addresses both gaps. Every derived quantity is expressed as an explicit function of the *independent primary measurements* and is differentiated through the data-reduction equations by the chain rule, so that the combined standard uncertainty is always formed as a root-sum-square over the “real” independent inputs. The shared-input correlations are then handled automatically and correctly, without introducing covariance terms by hand. The procedure is embedded directly in the post-processing code and is evaluated for every recorded point; it is demonstrated across the subsonic, transonic and supersonic regimes of a reference wind tunnel campaign, and is used to identify the dominant measurement source for each output. The emphasis throughout is on the methodology and its engineering relevance, not on the aerodynamic behavior of any particular model.

2. THEORETICAL FRAMEWORK

2. 1. LAW OF PROPAGATION OF UNCERTAINTY

Let a derived quantity y be computed from N measured quantities through a known data-reduction equation: $y = f(x_1, x_2, \ldots, x_N)$. If the inputs x_i are estimates with known standard uncertainties $u(x_i)$, a first-order Taylor expansion of f about the mean values of the inputs gives the combined variance of y as the first-order law of propagation of uncertainty [4, 5]:

$$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i, x_j), \tag{1}$$

where $\partial f/\partial x_i$ are the sensitivity coefficients and $u(x_i, x_j)$ is the covariance between x_i and x_j. When the inputs are independent, all covariance terms vanish and the combined standard uncertainty reduces to the root-sum-square of the individual contributions:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)}\,. \tag{2}$$

Equation (2) yields the standard (one-sigma) uncertainty. Where a stated level of confidence is required, an expanded uncertainty is obtained by multiplying the combined standard uncertainty by a coverage factor k [4, 8]:

$$U = ku_c(y), \tag{3}$$

with $k \approx 2$ corresponding to a coverage probability of about 95 % for an approximately normal output. In the present work, all results are reported as standard uncertainties (k = 1); but a coverage factor may be applied without changing the propagation itself.

2. 2. PROPAGATION THROUGH THE DATA REDUCTION CHAIN

The measurement system separates naturally into two layers. The first layer contains the *primary measurements*: quantities that are read directly from a calibrated sensor and whose standard uncertainties are therefore known a priori from the instrument specifications. The second layer contains the *derived quantities*, meaning flow parameters and aerodynamic coefficients obtained from the “primary variables” through the data-reduction equations. The primary variables are treated as mutually independent, since they originate from physically distinct transducers. The derived quantities, in contrast, are generally correlated because they are built from the same primary variables.

This distinction is the key to a consistent chain. A derived quantity is frequently obtained through one or more intermediate quantities: $x_i = x_i(z_1, \ldots, z_m)$, where z_k are the independent primary measurements. Applying the chain rule, the sensitivity of y with respect to each primary and the resulting combined uncertainty are:

$$\frac{\partial y}{\partial z_k} = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial x_i}{\partial z_k}, u_c^2(y) = \sum_{k=1}^{m} \left(\frac{\partial y}{\partial z_k} \right)^2 u^2(z_k). \tag{4}$$

Equation (4) is applied throughout this paper. Its practical consequence is simple but important: whenever an intermediate quantity is reused by several outputs, the propagation must be carried back to the primary measurements rather than stopped at the intermediate level. Stopping at the intermediate level and recombining intermediate uncertainties as if they were independent would ignore the correlation that the shared primary variables introduce, and would in general give an incorrect combined uncertainty. The recursive form $u(y) = |\partial y/\partial x|\ u(x)$ remains valid only when the intermediate x is the sole path through which the primary variables reach y; this special case is noted explicitly where it occurs.

Figure 1 shows the dependency graph of the reduction chain analyzed here. Three thermodynamic primary variables, total pressure p_0, static pressure p and total temperature T_0, are use to compute the Mach number and the static temperature, which in turn are used to calculate the speed of sound, viscosity, density, dynamic pressure and airspeed, and finally the Reynolds number. The balance loads and the model incidence are used for the aerodynamic coefficients through the dynamic pressure, while the pressure taps form an independent branch. The shared nodes M and T are precisely the quantities for which the primary-level expansion of equation (4) is required.

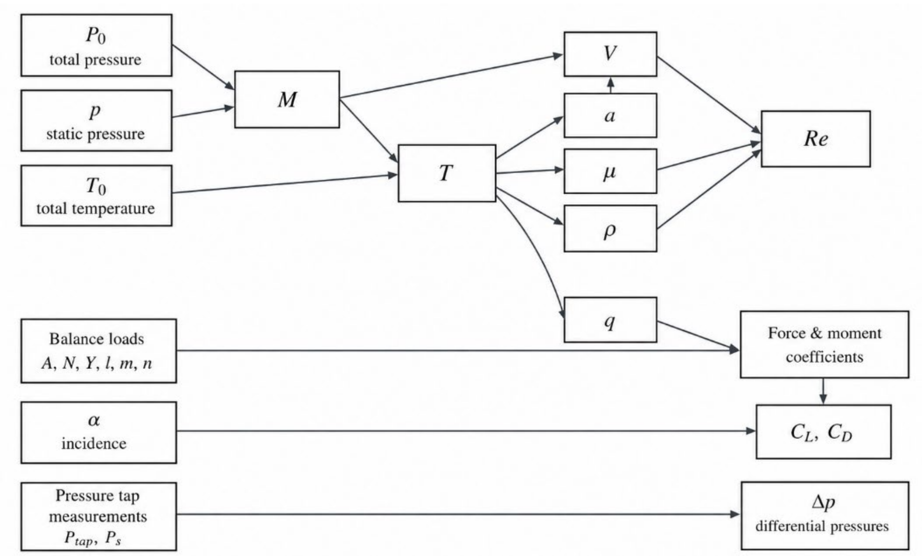


Fig. 1 – Propagation of standard uncertainty through the data-reduction chain.

The standard uncertainties of the primary measurements are listed in Table 1. The pressure and temperature values follow the calibrated accuracy of the facility transducers, whereas the balance force and moment channels follow the calibration of the strain-gauge balance. The incidence angle uncertainty corresponds to two arc-minutes of the angle-of-attack mechanism; and the base and cavity taps are differential channels referenced to the static-pressure manifold. These eleven entries are the only quantities whose uncertainty is assumed known. Every other uncertainty in the paper is derived from them through equation (4).

Table 1

Standard uncertainties of the primary measurements

Measured quantity	**Relative error [%]**	**Full scale**	**Standard uncertainty**
Flow total pressure, p_0	0.05	700 kPa	350 Pa
Flow static pressure, p	0.04	400 kPa	160 Pa
Base / cavity tap pressure, P_{tap}	0.10	344.74 kPa	344.73 Pa
Flow total temperature, T_0	–	–	0.2 K
Incidence angle, α	–	–	5.8×10^{-4} rad
Side force, Y	0.089	9 300 N	8.27 N
Normal force, N	0.055	14 000 N	7.70 N

Measured quantity	Relative error [%]	Full scale	Standard uncertainty
Axial force, A	0.355	2 500 N	8.87 N
Pitching moment, m	0.418	435 N·m	1.82 N·m
Rolling moment, l	0.564	590 N·m	3.33 N·m
Yawing moment, n	0.158	1 000 N·m	1.58 N·m

3. UNCERTAINTY MODEL OF THE MEASUREMENT CHAIN

The derived quantities are now treated in the order in which they appear in the chain. Each subsection follows the same three steps: the data-reduction equation is stated; its sensitivity coefficients with respect to the independent primary variables are obtained by the chain rule; and the combined standard uncertainty is assembled with equation (4). The thermodynamic constants are $\gamma = 1.4$ and $R = 287.05\ \mathrm{J{\cdot}kg^{-1}{\cdot}K^{-1}}$ for air. The viscosity constants are given in Section 3.6. The reference area S_{ref} and reference length L_{ref} are fixed geometric constants and are treated as exact, so they do not contribute to the uncertainty.

3. 1. MACH NUMBER

The Mach number is obtained from the ratio of total to static pressure under the isentropic assumption [12]:

$$M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}. \tag{5}$$

Since M depends only on p_0 and p, the sensitivity coefficients are:

$$\frac{\partial M}{\partial p_0} = \frac{(p_0 / p)^{\frac{\gamma-1}{\gamma}}}{\gamma M p_0},\quad \frac{\partial M}{\partial p} = -\frac{(p_0 / p)^{\frac{\gamma-1}{\gamma}}}{\gamma M p} \tag{6}$$

and the combined standard uncertainty follows from equation (2):

$$u_M = \sqrt{\left(\frac{\partial M}{\partial p_0}\right)^2 u_{p_0}^2 + \left(\frac{\partial M}{\partial p}\right)^2 u_p^2} = \frac{(p_0 / p)^{\frac{\gamma-1}{\gamma}}}{\gamma M}\sqrt{\left(\frac{u_{p_0}}{p_0}\right)^2 + \left(\frac{u_p}{p}\right)^2}. \tag{7}$$

The factor $1/M$ in the sensitivities shows that the Mach number becomes increasingly sensitive to pressure errors as M decreases, because at low subsonic speeds the pressure ratio p_0/p is close to unity and a small error in either pressure produces a comparatively large error in M.

3. 2. STATIC TEMPERATURE

The static temperature is not measured directly; it is recovered from the measured total temperature and the Mach number through the isentropic relation [12]:

$$T = \frac{T_0}{1+\frac{\gamma-1}{2}M^2}. \tag{8}$$

The direct sensitivity to T_0 and the sensitivity to the Mach number are:

$$\frac{\partial T}{\partial T_0} = \frac{1}{1+\frac{\gamma-1}{2}M^2}, \frac{\partial T}{\partial M} = -\frac{T_0(\gamma-1)M}{\left(1+\frac{\gamma-1}{2}M^2\right)^2}. \tag{9}$$

Because M itself is a function of p_0 and p, the chain rule transfers these into sensitivities with respect to the pressure primary variables:

$$\frac{\partial T}{\partial p_0} = \frac{\partial T}{\partial M}\frac{\partial M}{\partial p_0}, \frac{\partial T}{\partial p} = \frac{\partial T}{\partial M}\frac{\partial M}{\partial p}, \tag{10}$$

so that T depends on the three independent primary variables p_0, p and T_0, and its combined standard uncertainty is:

$$u_T = \sqrt{\left(\frac{\partial T}{\partial p_0}\right)^2 u_{p_0}^2 + \left(\frac{\partial T}{\partial p}\right)^2 u_p^2 + \left(\frac{\partial T}{\partial T_0}\right)^2 u_{T_0}^2}. \tag{11}$$

3. 3. SPEED OF SOUND AND AIRSPEED

The local speed of sound depends on the static temperature only:

$$a = \sqrt{\gamma R T}. \tag{12}$$

Since T is the sole variable through which the primary variables are used to compute a, the recursive form is valid here and the uncertainty of a follows directly from that of T:

$$\frac{\partial a}{\partial T}=\frac{a}{2T},\ u_a=\frac{a}{2T}u_T\,. \tag{13}$$

The airspeed is the product of the Mach number and the speed of sound:

$$V=Ma\,. \tag{14}$$

Here the recursive shortcut must not be used. The naive expression $u_V^2=(a\,u_M)^2+(M\,u_a)^2$ treats M and a as independent, whereas a depends on T, which depends on M. Therefore, the two factors share the pressure primary variables and are therefore correlated. Expanding V through the chain rule to the independent primary variables removes this difficulty:

$$\begin{gathered}\frac{\partial V}{\partial p_0}=a\frac{\partial M}{\partial p_0}+M\frac{\partial a}{\partial T}\frac{\partial T}{\partial p_0},\\ \frac{\partial V}{\partial p}=a\frac{\partial M}{\partial p}+M\frac{\partial a}{\partial T}\frac{\partial T}{\partial p},\\ \frac{\partial V}{\partial T_0}=M\frac{\partial a}{\partial T}\frac{\partial T}{\partial T_0}\end{gathered} \tag{15}$$

and the combined standard uncertainty is the root-sum-square over the three primary variables:

$$u_V=\sqrt{\left(\frac{\partial V}{\partial p_0}\right)^2u_{p_0}^2+\left(\frac{\partial V}{\partial p}\right)^2u_p^2+\left(\frac{\partial V}{\partial T_0}\right)^2u_{T_0}^2}\ . \tag{16}$$

3. 4. DENSITY

The density follows from the ideal-gas law applied to the static state:

$$\rho=\frac{p}{RT}\,. \tag{17}$$

The partial derivatives at constant temperature and with respect to temperature are:

$$\frac{\partial\rho}{\partial p}\Big|_T=\frac{1}{RT},\quad \frac{\partial\rho}{\partial T}=-\frac{p}{RT^2}\,. \tag{18}$$

The static pressure influences the density both directly and through the static temperature (via the Mach number), whereas p_0 and T_0 act only through T. Collecting the total derivatives with respect to the independent primary variables gives:

$$\frac{\partial\rho}{\partial p_0}=\frac{\partial\rho}{\partial T}\frac{\partial T}{\partial p_0},\ \frac{\partial\rho}{\partial p}=\frac{1}{RT}+\frac{\partial\rho}{\partial T}\frac{\partial T}{\partial p},\ \frac{\partial\rho}{\partial T_0}=\frac{\partial\rho}{\partial T}\frac{\partial T}{\partial T_0}, \tag{19}$$

where the static-pressure term retains both the direct contribution $1/(R\ T)$ and the indirect contribution through T. The combined standard uncertainty is then:

$$u_\rho=\sqrt{\left(\frac{\partial\rho}{\partial p_0}\right)^2 u_{p_0}^2+\left(\frac{\partial\rho}{\partial p}\right)^2 u_p^2+\left(\frac{\partial\rho}{\partial T_0}\right)^2 u_{T_0}^2}\ . \tag{20}$$

3. 5. DYNAMIC PRESSURE

The dynamic pressure is expressed through the isentropic relation in terms of the static pressure and the Mach number:

$$q=\frac{1}{2}\gamma p M^2. \tag{21}$$

As for the density, p contributes both directly and through M, while p_0 contributes only through M. The total sensitivities are:

$$\frac{\partial q}{\partial p_0}=\gamma p M\frac{\partial M}{\partial p_0},\quad \frac{\partial q}{\partial p}=\frac{1}{2}\gamma M^2+\gamma p M\frac{\partial M}{\partial p} \tag{22}$$

and the combined standard uncertainty, which depends on the two pressure primary variables only, is:

$$u_q=\sqrt{\left(\frac{\partial q}{\partial p_0}\right)^2 u_{p_0}^2+\left(\frac{\partial q}{\partial p}\right)^2 u_p^2}\ . \tag{23}$$

3. 6. DYNAMIC VISCOSITY

The dynamic viscosity of air is evaluated from the static temperature with Sutherland's law [13, 14], using μ_{ref} = 1.716×10^{-5} Pa·s, T_{ref} = 273.15 K and the Sutherland constant S = 110.4 K:

$$\mu=\mu_{\text{ref}}\left(\frac{T}{T_{\text{ref}}}\right)^{3/2}\frac{T_{\text{ref}}+S}{T+S}. \tag{24}$$

Viscosity depends on the primary variables through T alone, so the recursive form applies and its sensitivity and uncertainty are:

$$\frac{\partial\mu}{\partial T}=\mu\left(\frac{3}{2T}-\frac{1}{T+S}\right),\ u_\mu=\left|\frac{\partial\mu}{\partial T}\right|u_T\,. \tag{25}$$

3. 7. REYNOLDS NUMBER

The Reynolds number combines density, airspeed and viscosity with the reference length:

$$Re=\frac{\rho V L_{\mathrm{ref}}}{\mu}\,. \tag{26}$$

Density, airspeed and viscosity are not independent, but they all inherit the static temperature, hence the pressure and total-temperature primary variables. The propagation is therefore carried to the primary level: for each primary $z \in \{p_0, p, T_0\}$ the total sensitivity is

$$\frac{\partial Re}{\partial z}=\frac{L_{\mathrm{ref}}}{\mu}\left(V\frac{\partial\rho}{\partial z}+\rho\frac{\partial V}{\partial z}\right)-\frac{\rho V L_{\mathrm{ref}}}{\mu^2}\frac{\partial\mu}{\partial z},\ z\in\{p_0,p,T_0\}\,, \tag{27}$$

with $\partial\mu/\partial z = (\partial\mu/\partial T)(\partial T/\partial z)$, and the combined standard uncertainty is the root-sum-square over the three primary variables:

$$u_{Re}=\sqrt{\sum_{z\in\{p_0,p,T_0\}}\left(\frac{\partial Re}{\partial z}\right)^2 u_z^2}\,. \tag{28}$$

It is instructive to contrast equation (28) with the convenient relative-uncertainty form that would be obtained by treating ρ, V and μ as independent:

$$u_{Re}\,|_{\mathrm{indep}}=Re\sqrt{\left(\frac{u_\rho}{\rho}\right)^2+\left(\frac{u_V}{V}\right)^2+\left(\frac{u_\mu}{\mu}\right)^2} \tag{29}$$

Equation (29) neglects the correlations among ρ, V and μ that arise from their common dependence on the static temperature; it is retained here only for comparison, while equation (28) is the form actually used.

3. 8. AERODYNAMIC FORCE AND MOMENT COEFFICIENTS

The body-axis forces (A, N, Y) and moments (l, m, n) are measured by the balance and are non-dimensionalized with the dynamic pressure and the reference geometry:

$$C_A = \frac{A}{qS_{\text{ref}}},\quad C_N = \frac{N}{qS_{\text{ref}}},\quad C_Y = \frac{Y}{qS_{\text{ref}}},$$
$$C_l = \frac{l}{qS_{\text{ref}}L_{\text{ref}}},\quad C_m = \frac{m}{qS_{\text{ref}}L_{\text{ref}}},\quad C_n = \frac{n}{qS_{\text{ref}}L_{\text{ref}}}. \tag{30}$$

Each coefficient combines an independent balance channel with the dynamic pressure. Treating S_{ref} and L_{ref} as exact, the first-order propagation for a generic coefficient $C = F/(q\, S_{\text{ref}})$ – with the moment coefficients using $S_{\text{ref}}\, L_{\text{ref}}$ – is:

$$u_C = \sqrt{\left(\frac{u_F}{qS_{\text{ref}}}\right)^2 + C^2\left(\frac{u_q}{q}\right)^2}\ , \tag{31}$$

where u_F is the balance-channel uncertainty and u_q/q is the relative dynamic-pressure uncertainty from equation (23). The first term is the resolution of the balance referred to the load, whereas the second term is the share inherited from the freestream calibration through q. The wind-axis lift and drag coefficients are formed by rotating the body-axis coefficients through the incidence:

$$C_D = C_A\cos\alpha + C_N\sin\alpha, C_L = -C_A\sin\alpha + C_N\cos\alpha \tag{32}$$

and, propagating C_A, C_N and the incidence α (whose sensitivities are $\partial C_D/\partial\alpha = C_L$ and $\partial C_L/\partial\alpha = -C_D$):

$$u_{C_D} = \sqrt{\left(\cos\alpha\, u_{C_A}\right)^2 + \left(\sin\alpha\, u_{C_N}\right)^2 + \left(C_L u_\alpha\right)^2}\ , \tag{33}$$

$$u_{C_L} = \sqrt{\left(\sin\alpha\, u_{C_A}\right)^2 + \left(\cos\alpha\, u_{C_N}\right)^2 + \left(C_D u_\alpha\right)^2}\ . \tag{34}$$

3. 9. DIFFERENTIAL PRESSURE MEASUREMENTS

The base and cavity taps are differential channels reporting the difference between a tap pressure and the static-pressure reference:

$$\Delta P = P_{\text{tap}} - P_s\,. \tag{35}$$

Being the difference of two nominally independent measurements, the combined standard uncertainty of each channel is:

$$u_{\Delta P} = \sqrt{u_{P_{\text{tap}}}^2 + u_{P_s}^2}\ . \tag{36}$$

With the values of Table 1 (a tap-transducer uncertainty of 344.73 Pa and a static-reference uncertainty of 160 Pa) this gives a constant differential uncertainty:

$$u_{\Delta P} = \sqrt{(344.73)^2 + (160)^2}\ \text{Pa} \approx 380\,\text{Pa}\,, \tag{37}$$

which is independent of the flow regime, since both contributing uncertainties are fixed by the transducer full scales rather than by the local flow.

4. RESULTS AND DISCUSSION

The methodology of Sections 2 and 3 was applied point-by-point to every recorded sample of four representative runs covering the subsonic, transonic and supersonic regimes for a wind tunnel test campaign. For each derived quantity, the post-processing produces a nominal time history together with its standard-uncertainty band, evaluated from the primary uncertainties of Table 1. The discussion below summarizes the propagation behavior. The detailed figures show the nominal value (solid line) enclosed by the $\pm u$ band (shaded area).

4. 1. FLOW QUANTITIES ACROSS THE MACH RANGE

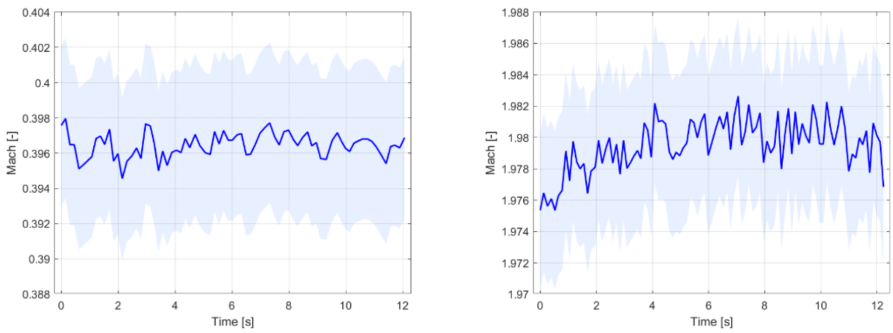
Table 2 collects the mean relative standard uncertainty of the eight derived flow quantities at the four regimes. Three qualitatively different behaviors are observed. The Mach number, airspeed, dynamic pressure and Reynolds number show their largest relative uncertainty in the subsonic case, fall to a minimum near the transonic regime, and rise again at the highest Mach number. The static temperature, speed of sound and viscosity increase monotonically with Mach number. The density increases steadily with Mach number, reaching the largest relative value of the set at M = 3.5.

Table 2

Mean relative standard uncertainty of the derived flow quantities [%]

Quantity	M = 0.4	M = 1.05	M = 2.0	M = 3.5
Mach number, M	1.15	0.24	0.25	0.37
Static temperature, T	0.10	0.11	0.23	0.52
Speed of sound, a	0.05	0.05	0.12	0.26
Airspeed, V	1.11	0.20	0.15	0.11
Density, ρ	0.12	0.17	0.54	1.30
Dynamic pressure, q	2.25	0.38	0.30	1.08
Dynamic viscosity, μ	0.08	0.09	0.21	0.57
Reynolds number, Re	1.20	0.28	0.28	0.64

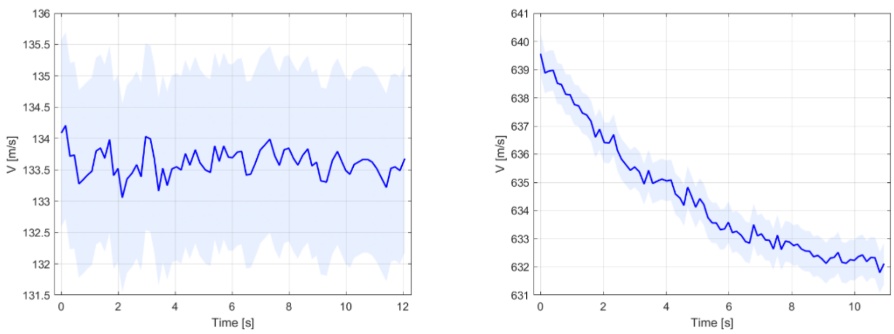
The Mach-number behavior follows directly from the $1/M$ factor in equation (6). At M = 0.4 the relative uncertainty reaches 1.15 %, the highest among the flow quantities, because the pressure ratio is close to unity and the fixed pressure errors of Table 1 are amplified. At supersonic speeds, the pressure ratio is large and the same errors translate into only 0.25–0.37%. Figure 2 illustrates the contrast between the wide subsonic band and the tight supersonic band.



a) M = 0.4 (subsonic) b) M = 2.0 (supersonic)

Fig. 2 – Mach-number standard uncertainty: a) subsonic, b) supersonic.

The airspeed variable also shows this trend, since $V = M\,a$ and the speed of sound is comparatively well determined. As shown in Figure 3, the relative uncertainty of V falls from 1.11 % at M = 0.4 to 0.11 % at M = 3.5, tracking the Mach number contribution almost exactly. It is here that the primary-level expansion of equations (15)–(16) matters: had M and a been treated as independent, the strong common dependence on the pressure primary variables would have been omitted.

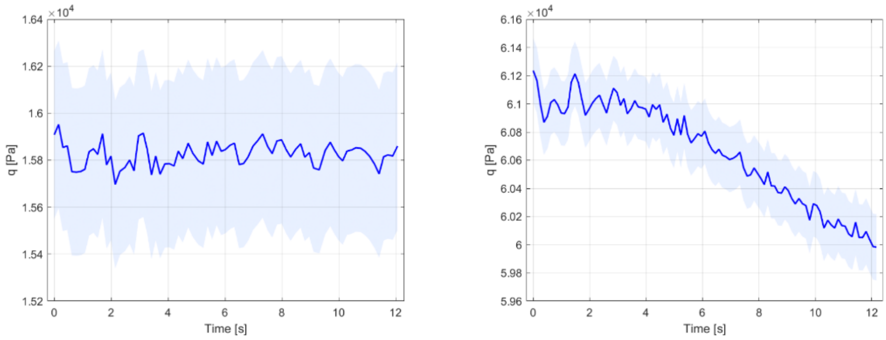


a) M = 0.4 b) M = 3.5

Fig. 3 – Airspeed standard uncertainty: a) subsonic, b) supersonic.

The static temperature, speed of sound and viscosity behave differently. Their relative uncertainties grow with Mach number because the static temperature becomes small and increasingly sensitive to the Mach number through equation (9). A useful internal check is that the relative uncertainty of the speed of sound is consistently about half that of the static temperature at every regime (0.05 % against 0.10 % at M = 0.4, 0.26 % against 0.52 % at M = 3.5), exactly as required by $a \propto \sqrt{T}$ and equation (13). This consistency confirms that the chain has been differentiated correctly.

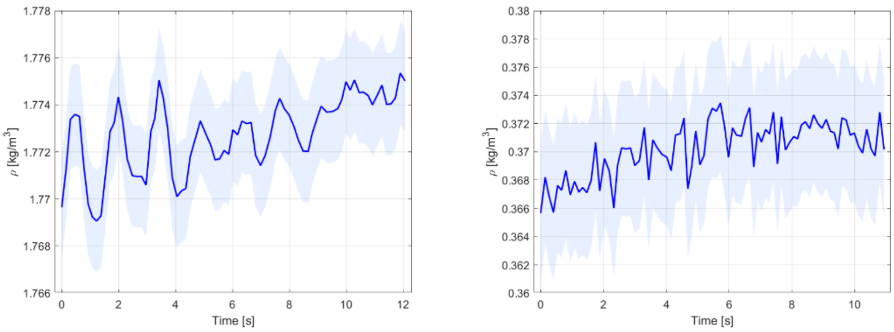
The dynamic pressure shows the strongest subsonic penalty of all flow quantities, 2.25 % at M = 0.4, because $q \propto M^2$ and the relative Mach uncertainty enters with a factor of two. As the Mach number increases, the dynamic pressure uncertainty falls to a transonic–supersonic minimum of about 0.3 % and then rises again to 1.08 % at M = 3.5, where the small static pressure of the high-supersonic test section inflates the relative static-pressure error (Figure 4).



a) M = 0.4 b) M = 1.05 (transonic)

Fig. 4 – Dynamic-pressure standard uncertainty: a) subsonic, b) transonic.

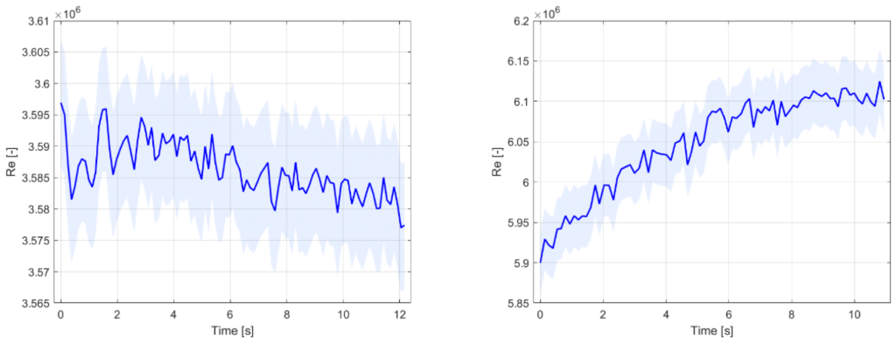
The density mirrors the static-pressure behavior. As the Mach number increases, the static pressure in the test section becomes lower and its fixed 160 Pa uncertainty grows in relative terms, so that the density uncertainty rises from 0.12 % at M = 0.4 to 1.30 % at M = 3.5 (Figure 5).



a) M = 0.4 b) M = 3.5

Fig. 5 – Density standard uncertainty: a) subsonic, b) supersonic.

The Reynolds number combines these effects through equation (28). It is largest in the subsonic case (1.20 %), where the airspeed term dominates, falls to a minimum of about 0.28 % at transonic and low-supersonic conditions, and rises to 0.64 % at M = 3.5, where the density term takes over (Figure 6). The Reynolds number is thus a sensitive composite indicator whose dominant error source migrates from the velocity branch at low speed to the density branch at high speed.



a) M = 1.05 b) M = 3.5

Fig. 6 – Reynolds-number standard uncertainty: a) transonic, b) supersonic.

4. 2. AERODYNAMIC COEFFICIENTS

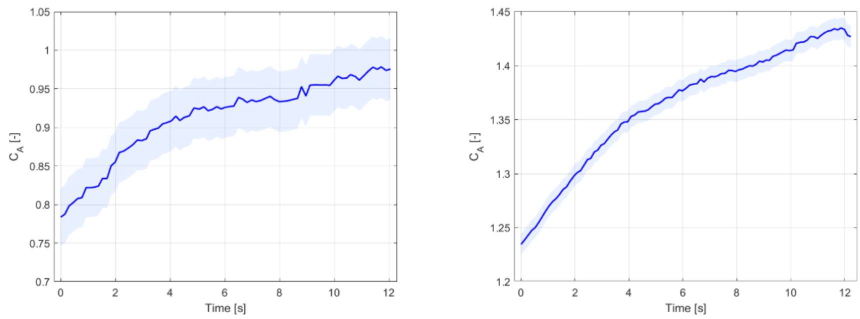
For the aerodynamic coefficients, the relative uncertainty is not always a meaningful descriptor, because several coefficients pass through zero during an incidence sweep, where the relative value diverges while the absolute uncertainty stays finite and well behaved. Table 3 therefore reports the mean absolute standard uncertainty, which is the proper measure for near-zero coefficients.

Table 3

Mean absolute standard uncertainty of the aerodynamic coefficients (× 10^{-3})

Coefficient	**M = 0.4**	**M = 1.05**	**M = 2.0**	**M = 3.5**
Axial force, C_A	39.0	9.9	9.9	15.9
Normal force, C_N	29.0	7.5	7.8	6.3
Side force, C_Y	30.9	8.1	8.4	6.6
Lift, C_L	30.1	7.8	8.1	7.7
Drag, C_D	38.1	9.7	9.7	15.2
Rolling moment, C_l	85.0	22.2	23.0	18.0
Pitching moment, C_m	46.4	12.1	12.6	9.9
Yawing moment, C_n	40.3	10.5	10.9	8.6

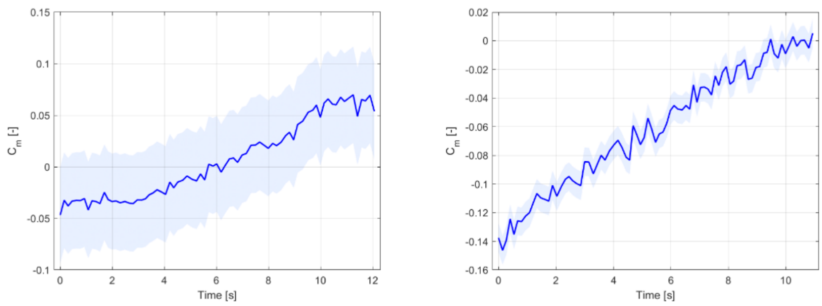
Two types of behavior are visible in equation (31). When the coefficient is large (as for the axial-force and drag coefficients), the second term, $C\ (u_q/q)$, dominates, so the coefficient uncertainty is governed by the dynamic pressure uncertainty. This produces the largest absolute values in the subsonic case (where u_q/q is largest and the loads are small relative to the balance full scale) and a secondary rise at M = 3.5 (where u_q/q increases again). Figure 7 shows this low-dynamic-pressure penalty for the axial-force coefficient: the band is markedly wider, in relative terms, at M = 0.4 than at M = 2.0.



a) M = 0.4 b) M = 2.0

Fig. 7 – Axial-force coefficient standard uncertainty: a) subsonic, b) supersonic.

When the coefficient is small (as for the normal-force, side-force and the three moment coefficients at these conditions), the second term nearly vanishes and equation (31) reduces to the balance term $u_F/(q\ S_{ref})$, which decreases monotonically as the dynamic pressure increases. The absolute uncertainties of these coefficients are therefore largest in the subsonic regime and decrease toward supersonic conditions, as Table 3 shows. Figure 8 illustrates the pitching-moment coefficient, whose nominal value is close to zero. The absolute band is essentially constant along each run and contracts with increasing dynamic pressure. This is consistent with the classical finding that, for lightly loaded channels, the uncertainty is dominated by the balance resolution scaled by the dynamic pressure, with the freestream calibration entering only weakly [2, 3].

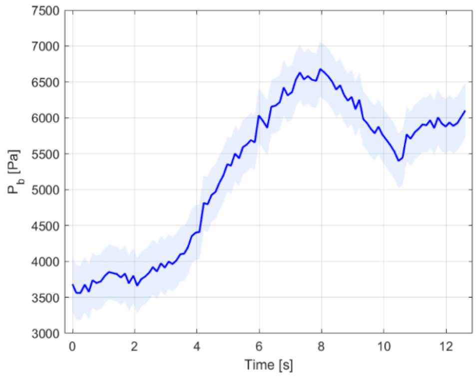


a) M = 0.4 b) M = 3.5

Fig. 8 – Pitching-moment coefficient standard uncertainty: a) subsonic, b) supersonic.

4. 3. DIFFERENTIAL PRESSURE MEASUREMENTS

The base and cavity taps follow equation (37): their absolute standard uncertainty is a constant of about 380 Pa, fixed by the transducer full scales and independent of the flow regime. Figure 9 shows a representative base-pressure channel, whose $\pm u$ band keeps a constant width while the measured pressure varies along the run. The relative uncertainty of a tap is therefore inversely proportional to the local pressure. It is small where the tap pressure is high and grows where the tap pressure is low, which at the higher Mach numbers (with their reduced static pressures) can make the relative tap uncertainty substantial even though the absolute value is unchanged.



M = 2.0 (representative base-pressure tap)

Fig. 9 – Base-pressure tap standard uncertainty (representative channel); the absolute band is constant along the run.

4. 4. DOMINANT ERROR SOURCES

Taken together, the results allow the dominant source to be assigned to each output. The total- and static-pressure transducers have the strongest influence on the Mach number, airspeed and dynamic pressure. The static-pressure transducer additionally influences the density at high Mach number. The total temperature probe, amplified by the Mach number at high speed, governs the static temperature, speed of sound and viscosity. The strain-gauge balance influences the lightly loaded coefficients, while the dynamic pressure uncertainty governs the heavily loaded ones. Finally, the differential taps are limited by a fixed transducer floor. Because the propagation is carried to the primary level for every point, this attribution is available continuously and can be used to target instrumentation upgrades where they reduce the reported uncertainty the most.

5. CONCLUSIONS

A structured chain-rule methodology has been presented for estimating the standard uncertainty of wind tunnel data within the post-processing methodology. Its central feature is that every derived flow parameter and aerodynamic coefficient is expressed as a function of the independent primary measurements and is differentiated through the data-reduction equations by the chain rule, so that the combined standard uncertainty is always a root-sum-square over independent inputs. This handles, automatically and correctly, the correlations introduced when intermediate quantities such as the Mach number and the static temperature are reused by several outputs.

Applied point-by-point across the subsonic, transonic and supersonic regimes, the method reproduces a coherent and physically interpretable picture. The Mach number-driven quantities are least certain at low subsonic speed, the temperature-driven quantities lose accuracy as the static temperature falls at high Mach numbers, the density and dynamic pressure are limited by the small static pressures of the supersonic test section, and the aerodynamic coefficients are governed either by the balance resolution or by the dynamic pressure uncertainty depending on the load level. Internal consistency checks, such as the speed of sound uncertainty being exactly half the static-temperature uncertainty, confirm the correctness of the differentiated chain.

Because the procedure is traceable, reproducible and evaluated for every recorded point, it turns uncertainty from an estimate done after post-processing into an integral output of data reduction, delivered alongside the reduced data and accompanied by a continuous attribution of the dominant error source. The framework is independent of any particular model and can be adopted directly as a reference for similar experimental campaigns. Its natural extensions are the inclusion of explicit correlation among balance channels and the cross-validation of the first-order estimates against a Monte-Carlo propagation of distributions.

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